Fast particle effects and microturbulence: stability, transport and size scaling

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Outline

- Motivation
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- Fusion $\alpha-$particles
- Collective $\alpha-$particle instabilities
- Nonpert. fast particle effects on global ITG Mode - linear gyrokinetic study
- Nonlinear global ITG/TEM Turbulence
- Diffusion, Size scaling of passive fast particle transport - Role of ITG/TEM Turbulence
- Conclusions

Motivation (1)

- Fast particle interaction with Tokamak bulk (thermals) has been investigated to a reasonable level. We have heard a great deal on the physics of MHD (bulk) + fast particles.

- It was generally understood that low frequency [$\omega/\omega_{ci} \ll 1$] microturbulence characterized by small perp. wavelength [$k_\perp \rho_i \simeq \hat{O}(1)$] and long parallel wavelengths [$k_{||}/k_\perp \ll 1$] such as ITG/TEM turbulence would not affect fast particles at all.
  - ▷ In TFTR Ohmic Discharges with neutral beams: "...Central Fast Ion Diffusion is an order of magnitude smaller than the thermal diffusivities... except during MHD activities when fast ions tend drive collective instabilities..."
    [W. W. Heidbrink et al PoFB (1991), W. W. Heidbrink & Sadler NF (1995)]
  - ▷ In TFTR DT Discharges : "...low radial diffusivities inferred for high energy alphas was consistent with orbit averaging over small scale turbulence"
    [S. J. Zweben et al NF (2001)]
  - ▷ 2D Hasegawa-Mima Turbulence simulation: "Orbit averaging" of small-scale turbulence over larger fast α orbits is the reason for $D_f/D_{th} \leq 0.1$
    [Myra et al, Phys. Fluids B 5, 1160 (1993), Manfredi et al, Phys. Rev. Lett, 76, 4360 (1996)]

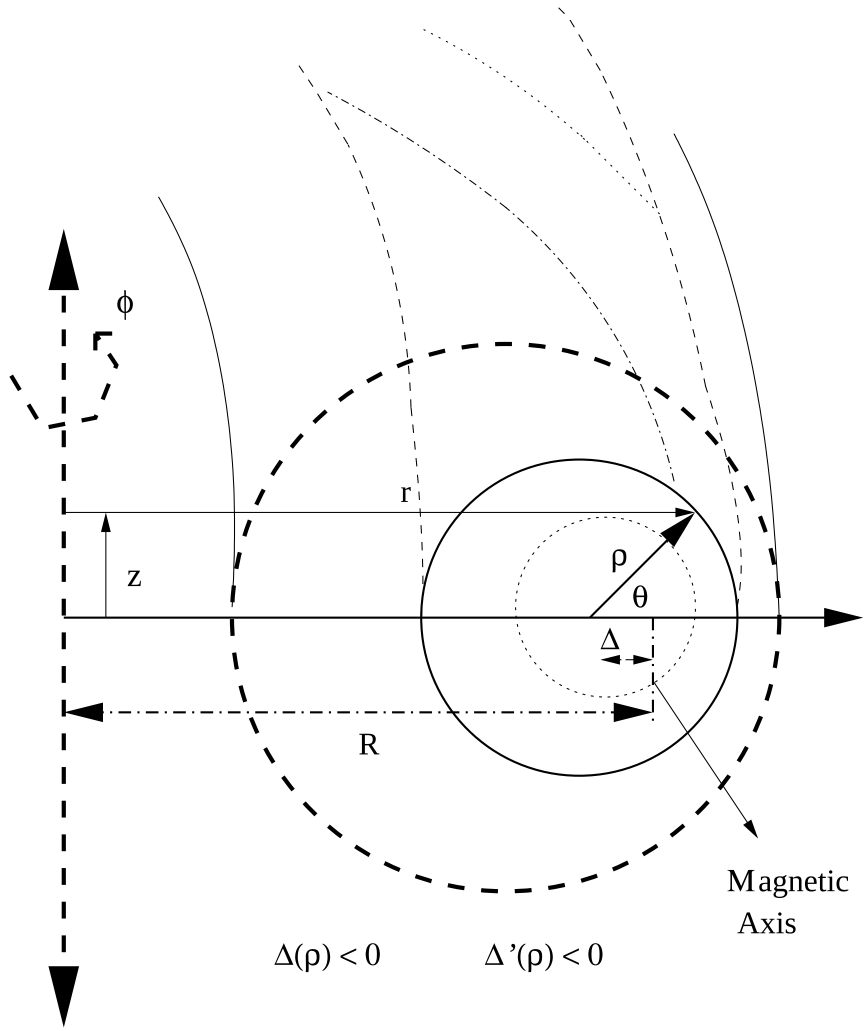
Motivation (2)

- More recently, the study of interaction of microturbulence and fast particles has been revived:
  - ▹ Strong correlations between suppression of ITG by energetic particles in ASDEX-U and formation of Ion Transport Barrier (or ITBs) in monotonic q-profile plasmas.
    [G. Tardini et al, Nuclear Fusion 47 (2007)]
    [M. Romanelli et al, Plasma Phys. Contr. Fusion 52 (2010)]
  - ▹ Redistribution of Neutral Beam Injected ions in MHD stable plasma
    [S. Gunter et al, Nuc Fusion 47 (2007)]
  - ▹ Redistribution of Neutral Beam Injected energetic ions by ITG Turbulence
    [W. W. Heidbrink et al, Phys. Rev. Lett. 103 (2009)]

- To properly understand this new situation, a model with proper FLR effects, Landau damping, magnetic resonances, transit resonances, drifts and radial equilibrium inhomogeneity with global effects would help.

Tokamak geometry - Background (1)

- Temperatures of $10 - 25\ KeV$ and density of more than few times $10^{20}\ m^{-3}$ have been achieved.
- Fusion products are born energetic (order of MeV)
- Heating is normally through an inductive process
- As resistivity decreases with temperature ($T^{-3/2}$) additional heating and current drive is necessary
- These methods involve either a resonant process (Cylocotron Heating) and nonresonant one (Neutral Beams)
- Both processes produce energetic or fast particles
- As a driven system, Tokamak has equilibrium density and temperture gradients.



Tokamak geometry - Background (2)

- Due to equilibrium gradients, a gyrating particle cannot "come back exactly" to the same location, resulting in cross-field "drifts".

- If $\Delta B/B \ll 1$, ΔB is variation on scale $\rho_{L,j}$ then following Kulsrud (1957), $\mu = v_\perp^2/2B$ is a "slowing varying" constant! (Magnetic Moment)

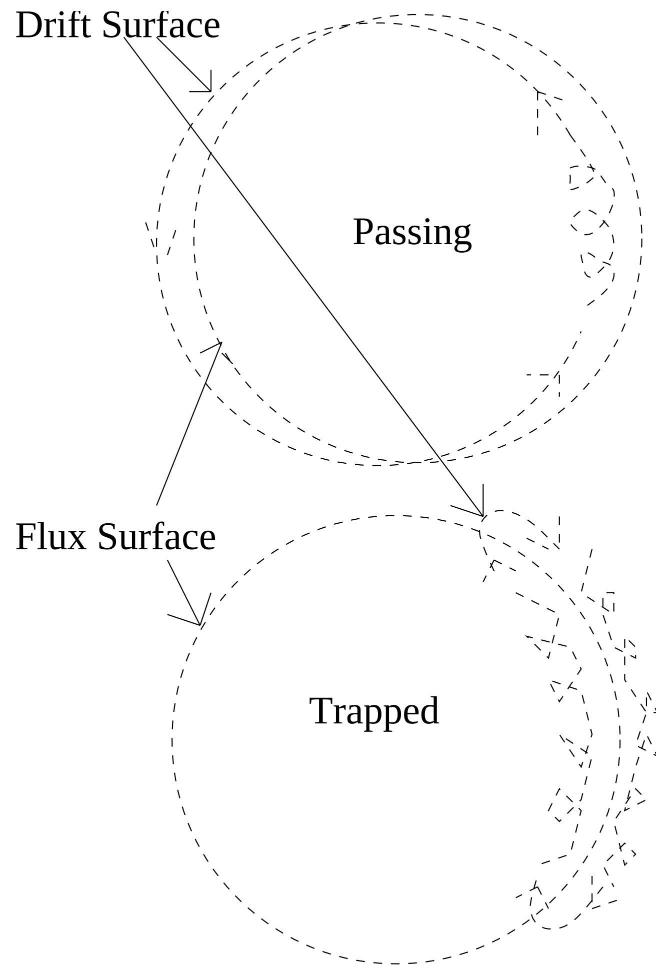
- When gyration is averaged out, particle guiding centers move along $\hat{e}_{||} = \vec{B}/B$ and drift across $\vec{B}$ with $\vec{v} \simeq \vec{v}_g = d\vec{R}/dt = v_{||}\hat{e}_{||} + \vec{v}_d +$ higher order. Here $\vec{v}_d = \vec{v}_{EB} + (v_\perp^2/2 + v_{||}^2)\hat{e}_z/(R\omega_{cj})$ contains $E \times B$, ∇B and curvature drifts.

Tokamak geometry - Background (3)

- Drift speeds are relatively small: $(v_{||}^2, v_\perp^2) \simeq v_{th,j}^2$, $v_{dj}/v_{th,j} \simeq 2v_{th,j}/(R\omega_{cj}) = \mathcal{O}(\rho_{L,j}/R)$ (non-MHD).
- For B-field with $B_{max,min}$, constancy of μ, ε, ψ_0 implies :

  Passing Particles: $0 < 1/(1+v_{||}^2/v_\perp^2) < B(\rho)/B_{max}$ with closed "drift-surface" in poloidal plane, $(r - R - v_{dj}/\omega_{tj})^2 + z^2 = \text{const}$, transit frequency $\omega_{t,j} = B_\theta v_{||}/B\rho = v_{||}/q(\rho)R$

  Trapped Particles: $B(\rho)/B_{max} < 1/(1+v_{||}^2/v_\perp^2) < B(\rho)/B_{min}$ with "banana-like" drift-surfaces in poloidal plane, $(\rho - \rho_0)^2 = v_{dj}^2/\omega_{bj}^2(1 - \theta^2/\theta_b^2)$, bounce-frequency $\omega_{bj} = v_\perp\sqrt{(}\rho/2R)/(q(\rho)R$, $\theta_b = v_{||0}\sqrt{(}2R/\rho)/v_\perp$



- Types of particles :
  [Co/Counter]Passing ions/electrons, Trapped ions/electrons, Fast Particles (Trapped,Passing)

Tokamak geometry - Background(4)

- Field line in a Tokamak equilibrium is a space curve which follows
  $r d\theta / B_\theta = R d\phi / B_\phi = dl / B$
- Following the field line around the torus we have
  $d\phi = r B_\phi / (R B_\theta) = q(r) d\theta$ where $q(r) = r B_\phi / (R B_\theta)$
- Following field lines ones around poloidal axis $(\Delta\theta = 2\pi)$, we follow q-times around toroidal dir.
- For rational q-value, a field line will map back on itself after q=m toroiidal transits. Hence not uniformly fill up a flux surface
- Because of axisymmetry, the fact that q is not usually an integer is not important. Flux surfaces can be defined for both rational or irrational values of q [KAM].
- For nonaxisymmetric perturbations, say, m_1 and n_1, $k_{||} = (n_1 * q - m_1)/(Rq)$ vanishes on surfaces if $m/n = m_1/n_1$ - called a mode rational surface.

A glance at MHD (1)

MagnetoHydroDynamics - For Bulk (Thermal) Particles

Simplification and Consequences

- Fluid description is assumed to be valid : Meaning, at each spatial point the velocities of individual particles within a small region (fluid element) at x is averaged and the resultant velocity is V_{fluid}

  - All particle vel. info. is lost! (Moments of Vlasov Eqn)

  - Vel $v(t)$ becomes $V_{fluid}(x,t)$ - a function of x!

  - Particle-based length/time scales such as $\rho_{L,j}$, $\omega_{c,j}$ (Larmor), $\omega_{trapped/passing,j}$, $\omega_{drift,j}$ (cross-field) are all lost!!

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V_{fluid}) = 0; \qquad \rho \frac{\partial V_{fluid}}{\partial t} + \rho V_{fluid} \cdot \nabla V_{fluid} = -\nabla P + J \times B;$$

$$\rho = mn, \qquad P/\rho^{\gamma} = \text{constant}$$

A glance at MHD (2)

- Entire plasma is considered as one single fluid and infinitely conducting

$$E_{fluid} = E + V_{fluid} \times B|_{lab} = \eta J, \qquad \eta = 0!$$

- Maxwell's Eqn

$$\frac{\partial B}{\partial t} = -\nabla \times E, \qquad \nabla \cdot B = 0 \qquad \nabla \times B = \mu_0 J$$

A glance at MHD (3)

- $E_{||} = 0$ always!

  $E_{||} = E \cdot B = -B \cdot V_{fluid} \times B = \eta J \cdot B = 0$

  Particle effects such as phenomenon of Landau Damping is eliminated!

- Frozen-in-Field Plasma : Due to infinte conductivity, any plasma motion (V_{fluid}) generates an induction Electric field E, such that there is no relative motion between V_{fluid} and B!

  $\frac{\partial B}{\partial t} - \nabla \times (B \times V_{fluid}) = \eta \nabla^2 B = 0$

  Ideal MHD model can not predict motion which grows (finite γ) and oscillates or rotates (finte ω_r) at the same time!

Fusion $\alpha-$particle (1)

- In a hot, dense enough plasma environment, Deuterium and Tritium fuse together to release Helium ion plus a neutron and a lot of kinetic energy.

- Neutrons carry away about $14.03\ MeV$ of energy (with which we won't bother ourselves)

- Helium ion (also called an α particle) comes away with $3.56\ MeV$

- From Tokamak fusion experiments [Nuc. Fusion, 32 (2) 1992], we know that in reactor-like situation, α's:
  - Will remain confined to self-heat the main plasma efficiently
  - Slowing down and energy transfer to the main plasma will be defined by classical Coulomb collisions with electrons and ions

- Since α's in a reactor-like situation are created by thermal Deuterium-Tritium reactions, it is expected that their birth profiles will be very broad as compared to today's experiments.

Fusion $\alpha-$particle (2)

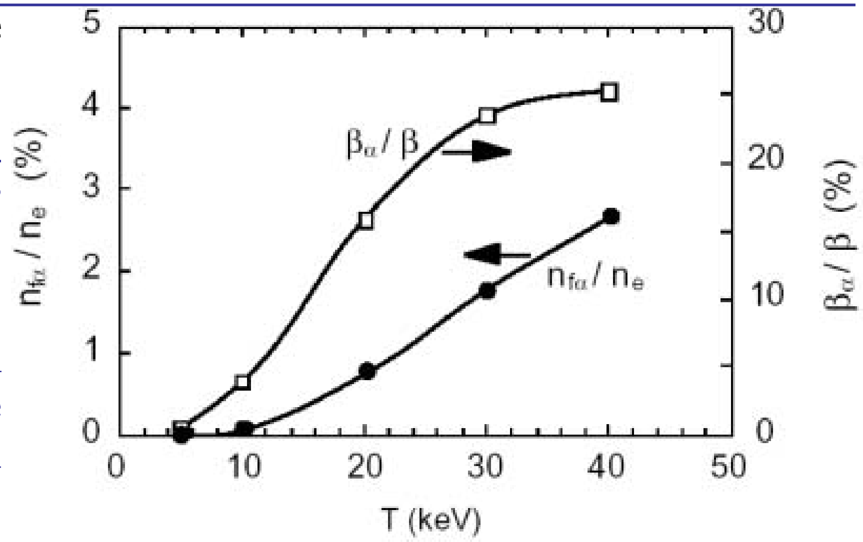
- Classical Interaction of α's with the main plasma
  - Stix [Plasma Physics 14, 367 (1972)] derived slowing-down rate of fast alpha energy E in a plasma:

    $\frac{dE}{dt} = -\frac{2E}{\tau_s}\left[1 + \frac{E_{crit}}{E}\right]^{3/2}$

    whereτ_s Spitzer slowing down time of α's with electron and E_{crit} is a critical energy value.
  - $\tau_s \propto \frac{T_e^{3/2}}{n}, \quad E_{crit} \propto T_e$
  - $E > E_{crit}$ α's loose energy to plasma electrons
  - At $E = E_{crit}$, energy loss on ions and electrons is equal.
  - $E < E_{crit}$ plasma ions gain energy from α's
  - In Tokamak experiments, these ideas have been confirmed [Taylor et al, Phys. Rev. Lett., 76, 2722 (1996)]

Fusion $\alpha-$particle (3)

- Projected α density and β profiles in a reactor-like Tokamak.
  - ▷ Classical slowing-down physics allows computation of local values of $n_{f\alpha}/n_e$ and β_α/β with $\beta = \beta_i + \beta_e + \beta_\alpha$ (total β) [Uckan et al, Fusion Tech., 13, 411 (1988)]
  - ▷ Assuming $T_e = T_i = T$, $Z_{eff} = 1.5$ then $n_{f\alpha}/n_e$ and β_α/β increase as temperature goes up. At $T_e 10.5\ KeV$ $\beta_\alpha/\beta \simeq 5\%$ and $n_{f\alpha}/n_e \simeq 0.1\%$. In the core $(20KeV)$, 0.8%, 15%!



(Local values) Scaling of alpha density and beta with T (Ti=Te, Z=1.5)
Uckan et al, Fusion Technology, 13, 411 (1988)

- Neoclassical Diffusion of α's
  - ▷ In the core, α's obey neoclassical diffusion and this has been experimentally verified with $D_{\alpha,core} \simeq 0.1\ m^2/s$ (10 times smaller than anomalous thermal ion diffusion!)
  - ▷ In the outer region, "ripple" losses give rise to rapid diffusion (ripple banana orbit)
  - ▷ "Orbit averaging" of small-scale turbulence over larger fast α orbits is the reason for $D_\alpha \simeq 0.1\ m^2/s$ [Myra et al, Phys. Fluids B 5, 1160 (1993), Manfredi et al, Phys. Rev. Lett, 76, 4360 (1996)]
  - ▷ Question is: "at reactor-scales, if turbulence scale size increases with machine size (non-gyro-Bohm), then, what happens to orbit averaging and D_α value?"

Collective $\alpha-$particle instabilities (1)

- Low-frequency MHD Modes
  - In a "gently" changing $B - field$, single particle energy W, Ist adiabatic invariant μ and angular momentum P_ϕ are conserved.
  - Consequently particle orbits in a Tokamak are "closed".
  - In a poloidal projection, trapped particles undergo "banana" and "potato" orbits. Later orbits are attributed to large FLR size compared to banana width (radially nonlocal).
  - Periodic motion of banana's (potato's) leads to precessional drift (frequency ω_D) which is proportional to W.
  - Depending on its energy range, trapped fast particle precessional frequency ω_{Df} can be larger than or comparable or smaller than typical ω of low-frequency MHD disturbances.

Collective $\alpha-$particle instabilities (2)

- ▷ If $\omega \ll \omega_{Df}$ then energetic banana orbits complete many toroidal revolutions during one ω^{-1} implying conservation of magnetic flux through the toroidal trajectory of the banana center (third adiabatic invariant). This has stabilizing effect. [Antonsen and Lee, Phys. Fluids, 25, 131 (1982)].

- ▷ If $\omega \simeq \omega_{Df}$, then third adiabatic invariant breaks down. Resonant destabilization results.

- ▷ Typically, if $\omega \simeq \omega_{*i}$ then typical fast particle energy $\simeq 100KeV$ comparable to standard auxillary drive energy range.

- **Internal KinK Mode + Fast Particles :**
  - ▷ $\omega \simeq \omega_{Df} \longrightarrow$ Destabilization of internal kink leading to "Fishbone instability" [McGuire et al, Phys.Rev.Lett., 60, 891 (1983)]
  - ▷ $\omega \ll \omega_{Df} \longrightarrow$ Stabilization of sawteeth period leading to "Monster sawteeth" [Porcelli, PPCF, 33, 1601 (1991), Campbell et al, Phys.Rev.Lett., 60, 2148 (1988)]

Collective $\alpha-$particle instabilities (3)

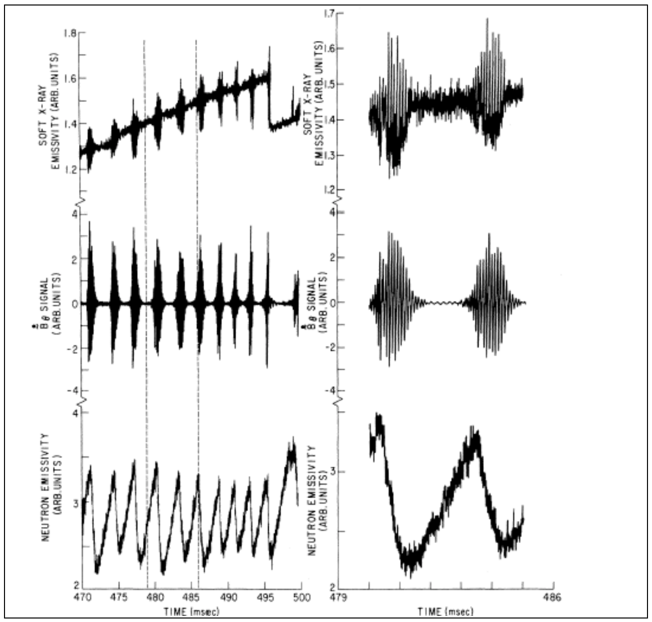
- High-n Ballooning Modes + Fast Particles :

  - ▷ $\omega \simeq \omega_{Df} \longrightarrow$ Destabilization of Ballooning Modes lead to birth of "High-n Fast Particle Driven Kinetic Ballooning Modes" [Chang et al, Phys. Rev. Lett. 76, 1071 (1996), Tsai et al, Phys. Fluids B 5, 3284 (1993), Zonca et al, PPCF, 38, 2011 (1996)]

  - ▷ $\omega \ll \omega_{Df} \longrightarrow$ Stabilization of Ballooning Modes [Connor et al, Joint Varenna-Grenoble Meet (1982), Rosenbluth et al, Phys. Rev. Lett. 51 1967 (1983)]

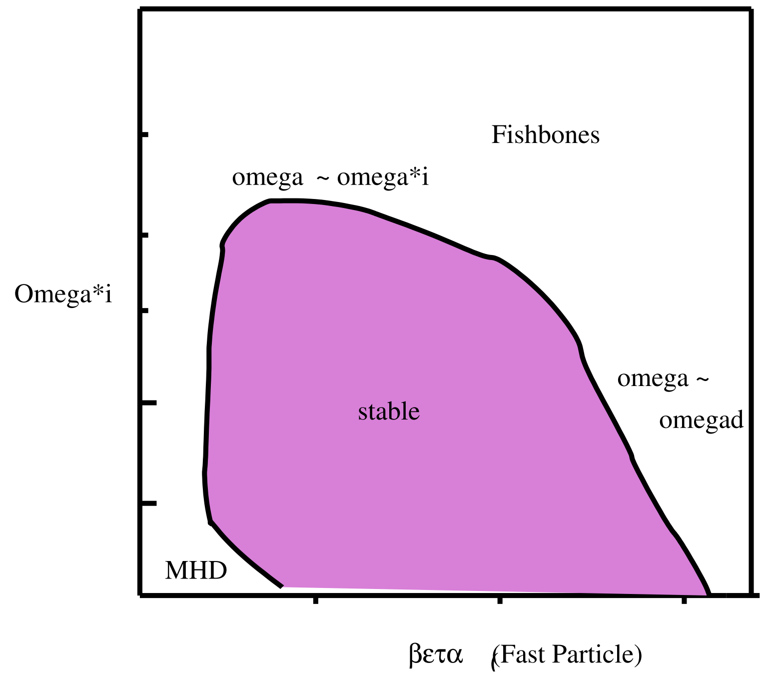
Collective $\alpha-$particle instabilities (5)

FISHBONES (experimental)



McGUIRE et al (1983) – FISHBONES

PHASE DIAGRAM

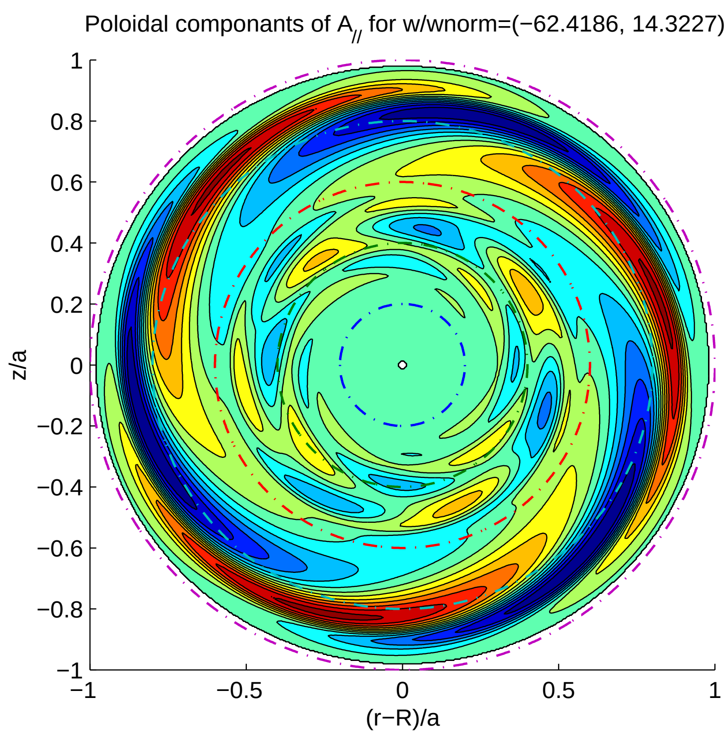


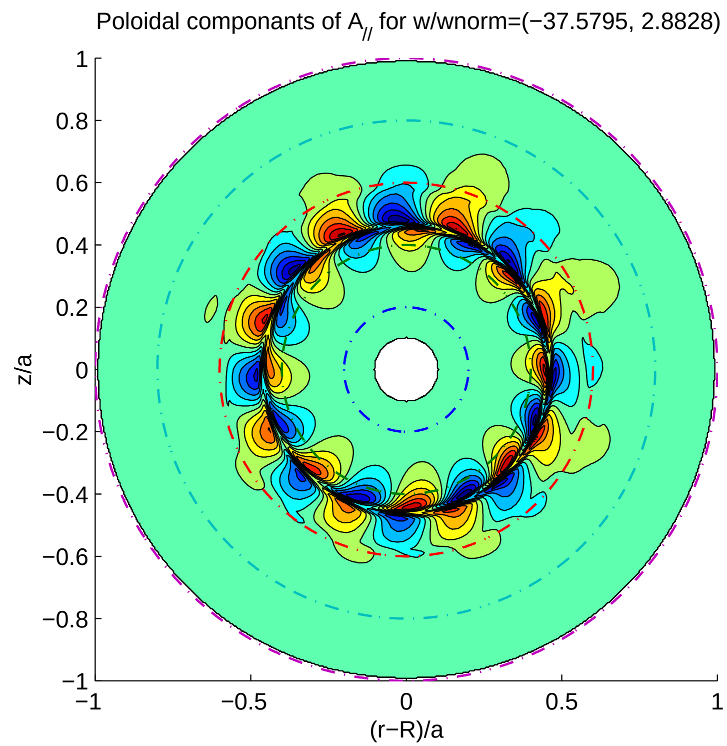
Coppi et al, Phys.Rev.Lett., 63,2733(1989)

Collective $\alpha-$particle instabilities (6)

- Low-frequency MHD Modes : Fishbone Instability and Monster Sawteeth

  (a) Mode structure of Kinetic Infernal Modes, $n = 3$ (left) and Standard KBM $n = 7$





- Low-n global Kinetic Ballooning Modes are shown to be unstable. [R. Ganesh et al, Phys. Rev. Lett. (2005)]
- Effect of Fast Particles on global modes such as Kinetic Infernal Modes and Low-n Kinetic Ballooning Modes is a open question.
- Multiple scales involved will necessitate novel numerical and mathematical approaches.

Collective $\alpha-$particle instabilities (7)

- Alfvenic Modes - Toroidal Alfven Eigenmode family
  - For reactor-like plasma parameters, $v_f/v_A(0) \simeq 1.9$ where v_f is the fast particle velocity and $v_A = B/\sqrt{\mu_0 \rho}$, What are the consequences?
  - In a slab-like system,
    Simple Alfven waves : $\vec{k}||\vec{B}_0$ and fluid vel. oscillates in the plane of $\vec{k}$ and $\vec{B}_0$.
    Shear Alfven waves : $\vec{k}$ at an angle to $\vec{B}_0$ and fluid vel. oscillates in the plane $\perp$ to $\vec{k}$ and $\vec{B}_0$. But along $\vec{B}_0$, $\omega = k_{||} v_A$ always
  - In a toroidal system, due to the effect of magnetic shear and toroidicity, shear Alfven stability problem can be written as

$$\frac{d}{dr}(\rho\omega^2 - F^2) r^3 \frac{d\vec{\xi}}{dr} - (m^2 - 1)[\rho\omega^2 - F^2] r\vec{\xi} + \omega^2 r^2 \frac{d\rho}{dr}\vec{\xi} = 0$$

where $F = (m - nq)B_\theta r/\sqrt{\mu_0}$, $m, n-$poloidal and toroidal mode numers, $q(r) = rB/RB_\theta$ is safety factor, $\vec{\xi}$ is the usual displacement vector and $\rho-$is mass density

[Cheng and Chance, Phys. Fluids, 29 (11), 3695, (1986),
J. Wesson, Tokamaks, 3rd Edition, pp: 402 (2004)]

Collective $\alpha-$particle instabilities (8)

- For a given ω, the coefficient of the highest derivative (in red) vanishes at the radius r for which

  $m - nq = \pm\frac{r\omega}{B_\theta/\sqrt{(\mu_0\rho)}} = \pm\frac{\omega}{2\omega_{TAE}}$ where $\omega_{TAE} = \frac{v_A}{2Rq}$.

- Singular solution $\longrightarrow$ no discrete spectrum, only continous spectrum

- Continuum spectra are always damped! Only discrete spectrum grows or damps

  In 2D Euler fluids, in 1967 K.M. Case obtained continuum spectra; similar idea has been applied to 2D Electron Plasmas by Corngold

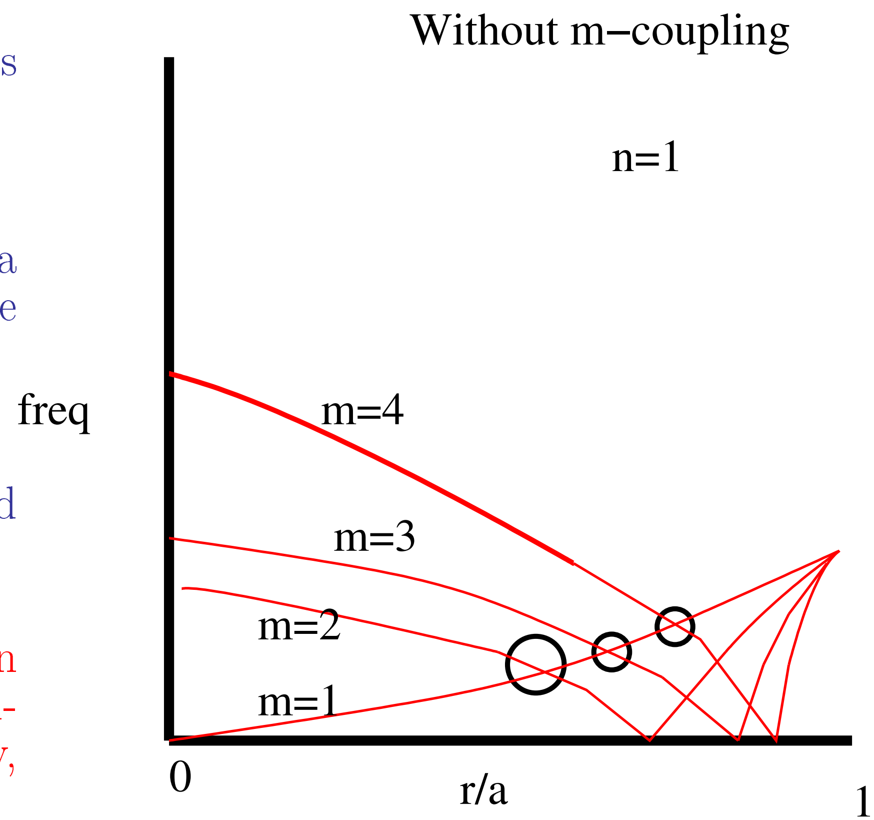
Collective $\alpha-$particle instabilities (9)

- In cylindrical geometry, for each (m, n), $\omega(r)$ is given by the above equation.

- In toroidal geometry, B-field is not uniform on a flux surface, i.e., poloidal Fourier component are strongly coupled.

- For a given n, $\omega(r)$ curves break and join and leave gaps in $\omega(r)$.

  Gap is analogous to the band-gap of an electron in a periodic well of crystal lattice [C. Kittel, Intro. to solid state physics,5th Edition, Weley, NewYork (1976)]



Collective $\alpha-$particle instabilities (10)

- Moreover, poloidal coupling introduces a discrete mode in the gap called "gap mode" or Toroidal Alfven Eigenmode (TAE).

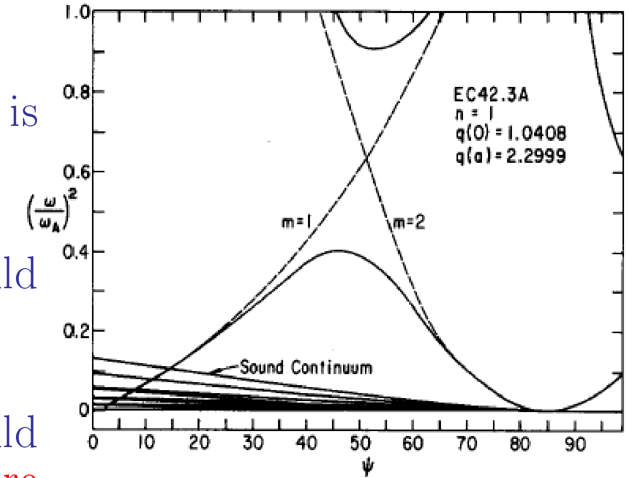
- While Shear Alfven Continuum is heavily damped, TAE is weakly damped and has $\omega \simeq \frac{v_A}{2qR} = \omega_{TAE}$.

- If fast particle velocity v_f is comparable to v_A then TAE could become resonantly unstable.

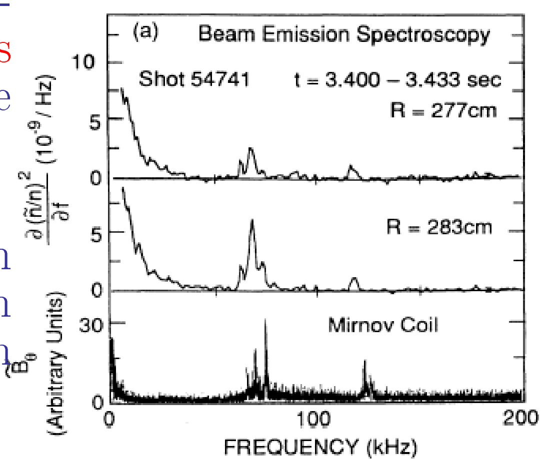
- Then, fast particles (fusion $\alpha-$particles) coupled to TAEs could then be violently thrown out of the plasma before they are "slowed-down", undermining the self-heating process

- (Top right figure) Cheng et al show the continuum ("degeneracy") lifted due to toroidal coupling and formation of "gap". [As an analogy, think of Stark-effect in quantum mechanics]. In the "gap" is a discrete mode which is TAE.

- (Bottom right figure)Fast particles (such as fusion alphas) can couple to TAE and drive them unstable. B_θ fluctuations then throws out fastparticles. This was shown experimentally in TFTR.



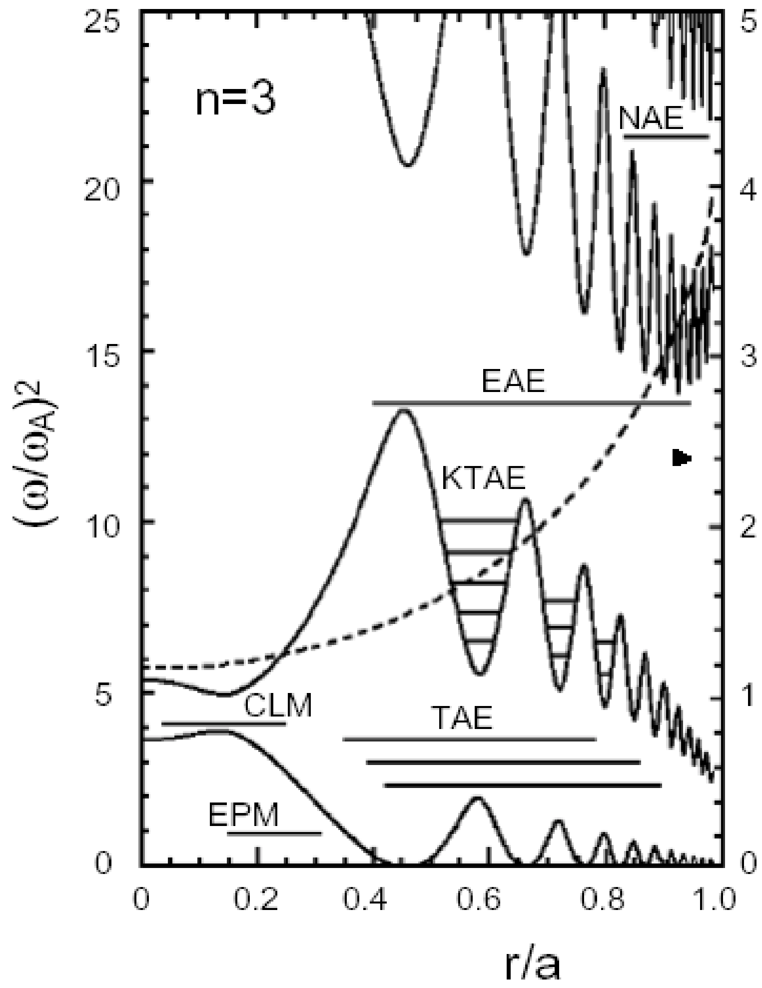
TAE – Cheng et al (1986)
Dashed lines are for uncoupled poloidal mode numbers



TAE Observation in TFTR. Wong et al (1991)

Collective $\alpha-$particle instabilities (11)

- As in Quantum Mechanics, more variations of TAE are born with more "asymmetries" or "forcing" : degeneracy (or continuum) splitting by "external forces"
- KTAE : Ion FLR and electron conductivity [Candy and Rosenbluth, Phys. Plasma 1, 356 (1995)]
- ETAE/NAE : Ellipticity/Triangularity induced split in the TAE continuum [Betti and Freidberg, Phys. Fluids B (3), 1865 (1992)]
- CLM : In the core, if $\epsilon > s$, then Core Localized TAE Mode is "lifted" out of degeneracy.
- For low-$n(1 < n10)$ TAEs, kinetic effects and non-linearity physics of TAEs in the presence of fast particles have been extensively studied by Berk, Breizmann, Pekker, Plasma Physics Reports, 23, 778 (1997)
- A cartoon of this zoo of TAEs is shown in the figure on the right.



TAE ZOO (Kramer et al 1998)

Collective $\alpha-$particle instabilities (12)

- Tiny $\tilde{B}_\theta$ fluctuations could derail a good percentage of energetic α.

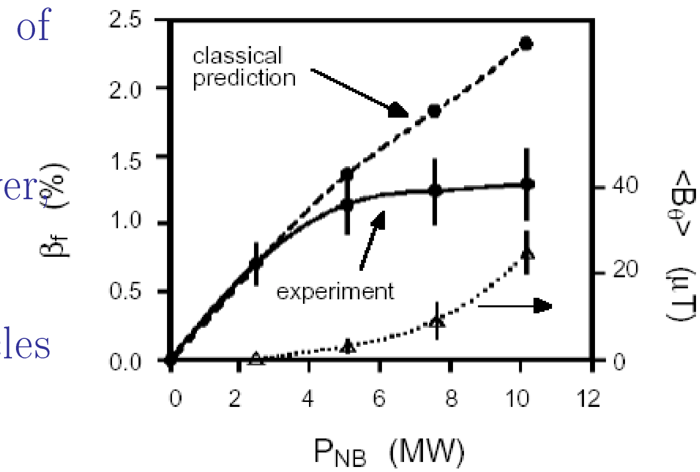
- (see figure on top right) Increase in fast particle power does not increase β_f according to classical theory.

- Fast-particle-destabilized-TAE removes fast-particles and saturates β_f.

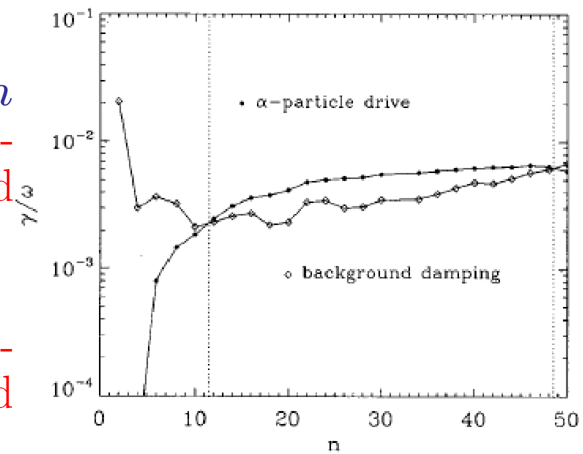
- Putvinski et al (1996) have predicted using a Linear Boundary Layer Code that for reactor-like machines n ranging in $10 < n < 50$ would be more important than low-n

- Nearly all of the present-day machines study low n $\alpha-$particle driven TAEs or its variations. In reactor-like situation, a TAE-turbulence could then be expected with $10 < n < 50$.

- As of now, very few models exist for studying interaction of TAE-continuum-decrete-mode-turbulence and fast particles.



TAE Mode Amplitude with NB heating power (DIII-D) (Strait, Heidbrink et al 1993)



For ITER-like parameters, linear alpha-driven TAEs Unstable for 10 < n < 50 (Putvinski et al 1996)

Global linear gyrokinetic theory (1)

- Aim of gyrokinetic theory is:
  - ▹ To describe effectively short perp. wavelength effects in tokamak, keeping FLR information to all orders
  - ▹ To efficiently describe low frequency waves (as compared to $\omega_{c,j}$) without resolving in time the Larmor motion
- Small parameters :
  - ▹ Ratio of Larmor radius to the Major radius or equilibrium gradient length scale $[\rho_{L,j}/R, \rho_{L,j}/L \ll 1]$
  - ▹ Ratio of freq. of plasma disturbance to the gyrofrequency $[\omega/\omega_{c,j} \ll 1]$

Global linear gyrokinetic theory (2)

- Vlasov Eqn for species j :

$$\frac{D}{Dt}f_j(\vec{r},\vec{v},t) \equiv \frac{\partial f_j}{\partial t} + \vec{v}\cdot\vec{\nabla} f_j + \frac{q_j}{m_j}(\vec{E}_T + \vec{v}\times\vec{B}_T)\cdot\vec{\nabla}_v f_j = 0$$

- $\vec{E}_T$ and $\vec{B}_T$ are total electric and magnetic fields to be obtained from Max. Eqns
- Assume an "equilibrium" without a zeroth order E-field and with zeroth order magnetic field $\vec{B}$.
- For small perturbation $\tilde{E}$ around this "equilibrium" one can expand $f_j = f_{0,j} + \tilde{f}_j$ such that $\tilde{f}_j/f_{0,j} \ll 1$
- Thus zeroth order eqn is :

$$\left.\frac{D}{Dt}\right|_{u.t.p.} f_{0j}(\vec{r},\vec{v},t) = 0 \quad where \quad \left.\frac{D}{Dt}\right|_{u.t.p.} \equiv \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla} + \frac{q_j}{m_j}(\vec{v}\times\vec{B})\cdot\vec{\nabla}_v$$

Global linear gyrokinetic theory (3)

- First order eqn is

$$\left.\frac{D}{Dt}\right|_{u.t.p.} \tilde{f}_j(\vec{r},\vec{v},t) = -\frac{q_j}{m_j}\tilde{E}\cdot\vec{\nabla}_{\vec{v}} f_{0j}$$

- Here "u.t.p." implies "unperturbed trajectory of particle" meaning equilibrium trajectories of particles

- Within a "linear" theory, the effect of perturbation does not "back react" and change the equilibrium features.

- Express $\tilde{E}$ in terms of $\tilde{\varphi}$, $\vec{B}$ in terms of $\vec{A}$, define change of variables $(\vec{r},\vec{v}) \rightarrow (\vec{r},\xi = v^2/2, \mu = v_\perp^2/2B, \psi_0)$. This helps express velocity degrees of freedom in terms of single particle constants of motion.

- Using particle canonical angular momentum for species j, i.e., $\psi_{0j} = \hat{e}_\phi \cdot \left[\vec{r}\times(\vec{A}+m_j\vec{v}/q_j)\right] = \psi + m_j r v_\phi/q_j$, one can write $f_{0j}(\vec{r},\vec{v}) = f_{0j}(\vec{r},\xi,\mu,\psi_{0j})$. Here cylindrical co-ordinates $\vec{r}\equiv(r,\phi,z)$ have been introduced and $\psi = rA_\phi$ is the poloidal flux function per unit radian. Such a transformation would enable one to express f_{0j} in terms of single particle constants of motion.

Global linear gyrokinetic theory (4)

- In the new variables, $\nabla_v f_{0j}$ term on the right hand side ($r.h.s$) of first order equation becomes

$$\nabla_v f_{0j}(r,\xi,\mu,\psi_{0j}) = v\left(1+\frac{m_j r v_\phi}{q_j}\frac{\partial}{\partial\psi_{0j}}\right)\frac{\partial f_{0j\psi}}{\partial\xi}+\frac{v_\perp}{B}\frac{\partial f_{0j\psi}}{\partial\mu}+\frac{m_j r\hat{e}_\phi}{q_j}\frac{\partial f_{0j}}{\partial\psi_{0j}}\bigg|_{\psi_0=\psi}$$

  where $f_{0j\psi}\equiv f_{0j}(\psi_{0j}=\psi)$ and $\hat{e}_\phi$ is the toroidal unit vector.

- Similarly using new variables, write perturbed distribution as "adiabatic" response and the "rest"!

$$\tilde{f}_j = h_j^{(0)} + \tilde{\varphi}\frac{q_j}{m_j}\left[\left(1-\frac{v_\phi}{\Omega_{pj}}\nabla_n\right)\frac{\partial f_{0j\psi}}{\partial\xi}+\frac{1}{B}\frac{\partial f_{0j\psi}}{\partial\mu}\right]$$

  Here $h_j^{(0)}$ is the zeroth order term of $h_j = h_j^{(0)} + \frac{1}{w_{cj}}h_j^{(1)} + \frac{1}{w_{cj}^2}h_j^{(2)}$..... Remember that we would like to describe modes with $\omega \ll \omega_{c,j}$ and note that $D/Dt \simeq O(\omega_{cj})$.

Global linear gyrokinetic theory (5)

- Putting the last two equations into first order eqn, we get:

$$\frac{D}{Dt}\bigg|_{u.t.p} h_j^{(0)}(r,v,t) = -\frac{q_j}{m_j}\left[\frac{\partial f_{0j\psi}}{\partial \xi}\frac{\partial}{\partial t} + \frac{v_{||}}{B}\frac{\partial f_{0j\psi}}{\partial \mu}\hat{e}_{||}\cdot\nabla + \frac{1}{\Omega_{pj}}\nabla_n f_{0j}\bigg|_{\psi}\hat{e}_{\phi}\cdot\nabla\right]\tilde{\varphi} + O($$

  In above equation, we have introduced the following definitions: $\Omega_{pj} = w_{cj}B_p/B$, $w_{cj} = q_j B/m_j$, $B_p = |\nabla\psi|/r$

- Gyroaveraging: In large aspect ratio Tokamak, $v = v_{\perp}(\hat{e}_{\varrho}cos\alpha + \hat{e}_{\theta}sin\alpha) + v_{||}\hat{e}_{||}$, where unit vectors $(\hat{e}_{\varrho}, \hat{e}_{\theta}, \hat{e}_{\phi})$ define the toroidal coordinates and α is the gyro-angle.

- We define gyro-averaging a quantity "Q" as

$$< Q >= \frac{1}{2\pi}\int_0^{2\pi} d\alpha Q(\alpha; ..)$$

Global linear gyrokinetic theory (6)

- In the perturbed eqn above, the terms in square brackets [..] on the $r.h.s.$ are all *equilibrium quantities* and are independent of α. Thus only the electrostatic potential is to be averaged. Similarly, on the left hand side ($l.h.s$), h_j^0 is independent of α, hence, only $D/Dt|_{u.t.p}$ is to be gyro-averaged.

- $$\left.\frac{D}{Dt}\right|_{u.t.p} \stackrel{gyro-averaging}{\Longrightarrow} \left.\frac{D}{Dt}\right|_{u.t.g} \equiv \frac{\partial}{\partial t} + (v_{||}\hat{e}_{||} + v_{dj}) \cdot \frac{\partial}{\partial R}$$

  where $v_{dj} = (v_{\perp}^2/2 + v_{||}^2)\hat{e}_z/(rw_{cj})$, $u.t.g.$ implies *unperturbed trajectory of guiding centers* R defined by $R = r + v \times \hat{e}_{||}/w_{cj}$.

- Similarly the electrostatic potential is to be gyroaveraged, but we dont know the form of ϕ!

  $$< \tilde{\varphi} >= \frac{1}{2\pi}\int_0^{2\pi} d\alpha \, [\, \tilde{\varphi}(r[\alpha], t)) \,] \Big|_{r=R-v\times\hat{e}_{||}/w_{cj}}$$

  Since $\tilde{\varphi}(r[\alpha], t)$ is an unknown function, the gyro-averaging is performed by first Fourier decomposing these functions.

Global linear gyrokinetic theory (7)

- Now represent the particle co-ordinate r by gyro-center R and remember that

$$J_p(x) = \frac{1}{2\pi}\int_0^{2\pi} d\alpha \exp[\iota(x sin\alpha - p\alpha)]$$

- The final form of gyrokinetic eqn is

$$\left.\frac{D}{Dt}\right|_{u.t.g} h_j(R,v,t) = -\left(\frac{q_j}{m_j}\right)\left[\frac{\partial f_{0j\psi}}{\partial \xi}\frac{\partial}{\partial t} + \frac{v_{||}}{B}\frac{\partial f_{0j\psi}}{\partial \mu}\hat{e}_{||}\cdot\nabla + \frac{1}{\Omega_{pj}}\nabla_n f_{0j}\right|_{\psi}\hat{e}_\phi\cdot\nabla\Bigg]\times$$
$$(\tilde{\varphi}(k;)J_0(k_\perp \varrho_{Lj}))$$
$$+O(\epsilon)$$

- Solution to the last eqn can be obtained by *Green function technique*: Replace the $r.h.s.$ by a unit source. For a Sinusoidal time dependence, solve for the Green function or Propagator $\mathcal{P}$. An explicit analytical form is obtainable by the characteristics of unperturbed trajectories of the guiding centre and perturbation theory for velocity.

Global linear gyrokinetic theory (8)

- Note that for a unit source, $\mathcal{P}$ is only dependent on "equilibrium quantities"!
- This situation can be further simplified by choosing a simple distribution function, for example one without μ or pitch angle dependence.
- Assume for equilibrium f_{0j}, a local Maxwellian of the form

$$f_{0j}(\xi,\mu,\psi) = f_{Mj}(\xi,\psi) = \frac{N(\psi)}{\left(\frac{2\pi T_j(\psi)}{m_j}\right)^{3/2}} \exp\left(-\frac{\xi}{T_j(\psi)/m_j}\right)$$

so that $\partial f_{0j}/\partial\mu \equiv 0$ by choice and density profile $N(\psi)$ is independent of the species type j.

- In terms of $\mathcal{P}$ solution to h_j^0 is in guiding center co-ordinates $\vec{R}$ is :

$$h_j^0(\vec{R},\vec{v},\omega) = -\left(\frac{q_j F_{Mj}}{T_j}\right)\int d\vec{k}\exp\left(\iota\vec{k}\cdot\vec{R}\right)\left(\omega-\omega_j^*\right)\left(\iota\,\mathcal{P}_j\right)\tilde{\varphi}(\vec{k};)J_0(k_\perp\varrho_{Lj}) + O(\epsilon$$

Global linear gyrokinetic theory (9)

- $\vec{k} = \kappa\ \hat{e}_\rho + k_\theta\ \hat{e}_\theta + k_\phi\ \hat{e}_\phi$ and $\kappa = (2\pi/\Delta\rho)\ k_\rho$, with $\Delta\rho = \rho_u - \rho_l$ which defines the radial domain, $k_\phi = n/r$ and $k_\theta = m/\rho$; ω is the *eigenvalue* and $\omega_j^* = \omega_{nj}\left[1 + \frac{\eta_j}{2}\left(\frac{v_{||}^2}{v_{thj}^2} - 3\right) + \frac{\eta_j v_\perp^2}{2\ v_{thj}^2}\right]$ with $\omega_{nj} = (T_j \nabla_n \ln N k_\theta)/(q_j B)$ is the *diamagnetic drift frequency*; $\eta_j = (d\ln T_j)/(d\ln N)$.

- Note also that since the large aspect ratio equilibria considered are axi-symmetric, the toroidal mode number "n" can be fixed and the problem is effectively two dimensional in (ρ, θ) (configuration space) or (κ, k_θ) (Fourier space).

- To obtain the particle density fluctuation $\tilde{n}_j(\vec{r};\omega)$, one needs to go from guiding center $(g.c.)$ co-ordinate $\vec{R}$ to *particle co-ordinate* $\vec{r}$ using $\vec{R} = \vec{r} + \vec{v} \times \hat{e}_{||}/w_{cj}$, by replacing h_j using the adiabatic relationship discussed earlier, followed by the integration over $\vec{v}$ keeping in mind the *gyro-angle* integration over α. This last integration on α yields an additional Bessel function "J_0" for $\tilde{\varphi}$, Thus, in real space $\vec{r}$, for species j, we finally have:

$$\tilde{n}_j(\vec{r};\omega) = -\left(\frac{q_j N}{T_j}\right)\left[\tilde{\varphi} + \int d\vec{k} \exp\left(\iota\vec{k}\cdot\vec{r}\right)\ \int d\vec{v}\frac{f_{Mj}}{N}\left(\omega - \omega_j^*\right)\left(\iota\mathcal{P}_j\right)\tilde{\varphi}(\vec{k};)J_0^2(x_{Lj}$$

Global linear gyrokinetic theory (10)

- The solution $\mathcal{P}$ for a given $(\vec{k},\omega)$ is simply

$$\mathcal{P}(\vec{R},\vec{k},\epsilon,\mu,\sigma,\omega) = \int_{-\infty}^{t} dt' \exp\left(\iota\left[\vec{k}\cdot(\vec{R}'-\vec{R}) - \omega t'\right]\right)$$
$$= \int_{-\infty}^{t} dt' \exp\left(\iota\int^{t'} dt''\vec{k}\cdot\vec{v}_g(t'') - \iota\omega t'\right) \qquad (3)$$

where guiding center velocity $d\vec{R}/dt = \vec{v}_g = \vec{v}_{||} + \vec{v}_d$ and $\vec{R}(t)$ is to be obtained by solving for guiding center trajectories as an "initial value problem" in equilibrium considered above. This is done by first assuming that the cross-field drift terms $[\vec{v}_d]$ to be small and drop them at the zeroth order and to include them iteratively at the next order.

- This procedure gives us $\mathcal{P}$:

$$\iota\mathcal{P} = \sum_{p,p'} \frac{J_p(x^{\sigma}_{tj})J_{p'}(x^{\sigma}_{tj})}{\omega - \sigma k_{||}v_{||} - p\omega_t} \exp(\iota(p-p')(\theta-\bar{\theta}_{\sigma})) \qquad (4)$$

Global linear gyrokinetic theory (11)

- Here $x^{\sigma}_{tj} = k_{\perp}\xi_{\sigma}$, $\xi_{\sigma} = v_d/\omega_t, v_d = \left(v_{\perp}^2/2 + v_{||}^2\right)/(\omega_c R), \omega_t = \sigma v_{||}/(q(s)R), \sigma = \pm 1$ (sign of $\vec{v}_{||}$), $k_{\perp} = \sqrt{\kappa^2 + k_{\theta}^2}$, $k_{||} = \left[nq(s) - m\right]/(q(s)R)$ and $\bar{\theta}_{\sigma}$ is defined as $\tan\bar{\theta}_{\sigma} = -\kappa/k_{\theta}$ and $s = \rho/a$, $a-$is the minor radius.

- A few points to be noted here: (1) Note that the grad-B and curvature drift effects appear through the argument of Bessel functions ($x^{\sigma}_{tj} = k_{\perp}v_d/\omega_t$) of the Propagator. Thus for example, "radial and poloidal coupling" vanishes if $x^{\sigma}_{tj} = 0$ in for Propagator and one would arrive at "cylindrical" results. Hence in our model, Bessel functions in propagator bring about coupling between neighbouring flux surfaces and also couple neighbouring poloidal harmonics. (2) Argument of Bessel functions J_p's in Propagator solution is i.e., $x^{\sigma}_{tj} = k_{\perp}\xi_{\sigma}$ also depends on transit frequency ω_t, x^{σ}_{tj} can become $x_{tj} \simeq \mathcal{O}(1)$. Hence transit harmonic orders are to be chosen accordingly.

- In this form $\mathcal{P}$ contains effects such as transit harmonic and its coupling, parallel velocity resonances (Landau), poloidal mode coupling.

- Similar propagators can be constructed for trapped particles as well.

Global linear gyrokinetic theory (12)

- Quasineutrality condition yields the "closure".

$$\sum_j \tilde{n}_j(r;\omega) \simeq 0; \tag{5}$$

- Now, putting back the density fluctuations in the quasineutrality condition and fourier transforming yields a Convolution Matrix due to equilibrium inhomogeneity.

- $$\sum_{\vec{k}'} \sum_{j=i,e,f} \mathcal{M}^j_{\vec{k},\vec{k}'} \; \tilde{\varphi}_{\vec{k}'} = 0$$

  where $\vec{k} = (\kappa, m)$ and $\vec{k}' = (\kappa', m')$. Note that we could have 3 species: passing ions (i), passing electrons (e) and fast ions (f) or more. In the following we discuss in detail the formulation for passing species.

- Also, $\vec{k} = (\kappa, m)$ and $\vec{k}' = (\kappa', m')$. With the following definitions, $\Delta\rho = \rho_u - \rho_l$ (upper and lower radial limits), $\Delta_\kappa = \kappa - \kappa'$ and $\Delta_m = m - m'$ matrix elements are :

Global linear gyrokinetic theory (13)

- Matrix elements are :

$$\mathcal{M}^{i}_{\vec{k},\vec{k}'} = \frac{1}{\Delta\rho}\int_{\rho_l}^{\rho_u} d\rho \exp(-\iota\Delta_\kappa\rho) \times \left[\alpha_p\delta_{mm'} + \exp(\iota\Delta_m\bar{\theta})\sum_p \hat{I}^0_{p,i}\right]$$

$$\mathcal{M}^{e}_{\vec{k},\vec{k}'} = \frac{1}{\Delta\rho}\int_{\rho_l}^{\rho_u} d\rho \exp(-\iota\Delta_\kappa\rho) \times \left[\frac{\alpha_p}{\tau(\rho)}\delta_{mm'} + \frac{\exp(\iota\Delta_m\bar{\theta})}{\tau(\rho)}\sum_p \hat{I}^0_{p,e}\right] \quad (6)$$

$$\hat{I}^l_{p,j} = \frac{1}{\sqrt{2\pi}v^3_{th,j}(\rho)}\int_{-vmax_j(\rho)}^{vmax_j(\rho)} v^l_{||}dv_{||}\exp\left(-\frac{v^2_{||}}{v^2_{th,j}(\rho)}\right)\left\{\frac{N^j_1 I^\sigma_{0,j} - N^j_2 I^\sigma_{1,j}}{D^{\sigma,j}_1}\right\}_{p'=p-}$$

- Velocity Space Integrals are:

$$I^\sigma_{n,j} = \int_0^{v_{\perp max,j(\rho)}} v^{2n+1}_\perp dv_\perp \exp\left(-\frac{v^2_\perp}{2v^2_{th,j}(\rho)}\right) J^2_0(x_{Lj})J_p(x'^\sigma_{tj})J_{p'}(x'^\sigma_{tj})$$

Global linear gyrokinetic theory (14)

- The definitions for Vel. Integrals: $v_{\perp max,j(\rho)} = min(v_{||}/\sqrt{\epsilon}, v_{max,j})$ which is "trapped particle exclusion" from ω independent perpendicular velocity integral $I^{\sigma}_{n,j}$; $\alpha_p = 1-\sqrt{\epsilon/(1+\epsilon)}$ is the fraction of passing particles; $\hat{I}^l_{p,j}$, is $\omega-dependent$ parallel integrals; $x^{\sigma}_{tj} = k_{\perp}\xi_{\sigma}$, $N^j_1 = \omega - w_{n,j}\left[1 + (\eta_j/2)(v^2_{||}/v^2_{th,j}) - 3)\right]$; $N^j_2 = w_{n,j}\eta_j/(2v^2_{th,j})$ and $D^{\sigma,j}_1 =< w_{t,j}(\rho) > (nq_s - m'(1-p)(\sigma v_{||}/v_{th,j}) - \omega$ where $< w_{t,j}(\rho) >= v_{th,j}(\rho)/(rq_s)$ is the average *transit frequency* of the species j.

- As integrals $I^{\sigma}_{n,j}$ are independent of ω and dependent only on $v_{\perp}$, σ and other equilibrium quantities, one may choose to calculate and store them as interpolation tables (memory intensive) or alternatively, one may choose to calculate them when needed (CPU-time intensive).

- Various numerical convergence tests should be performed with number of radial and poloidal Fourier modes, equilibrium profile discretization and velocity integrals.

Global linear gyrokinetic theory (15)

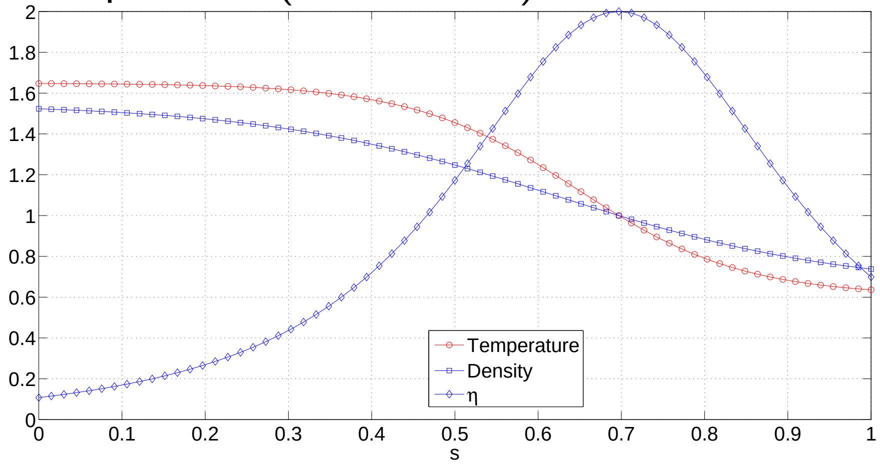
- Linear gyrokinetic eqns is formally solved using the equilibrium trajectories of particles.
- As the drift excursions are of $\mathcal{O}(\rho_{L,j}/R_0)$, a perturbative solution for guiding centre drift yields analytical solution for the Propagators (unit source solution) for both passing and trapped particles (not shown, but the method is the same!)
- This solution depends only on equilibrium quantities!
- Spatial inhomogeneity introduces coupling in spectral space $[\vec{k}]$.
- Model includes fully nonadiabatic ions, electrons and fast particles - all at the same physics footing!. This becomes possible because its a linear, spectral approach in space and time. Electrons and ions are not "pushed" in time.
- Particles which are deeply trapped or deeply passing are treated correctly. Model doesn't account for particles near the passing-trapping border in vel-space, as it is hard to obtain analytical equilibrium trajectories.

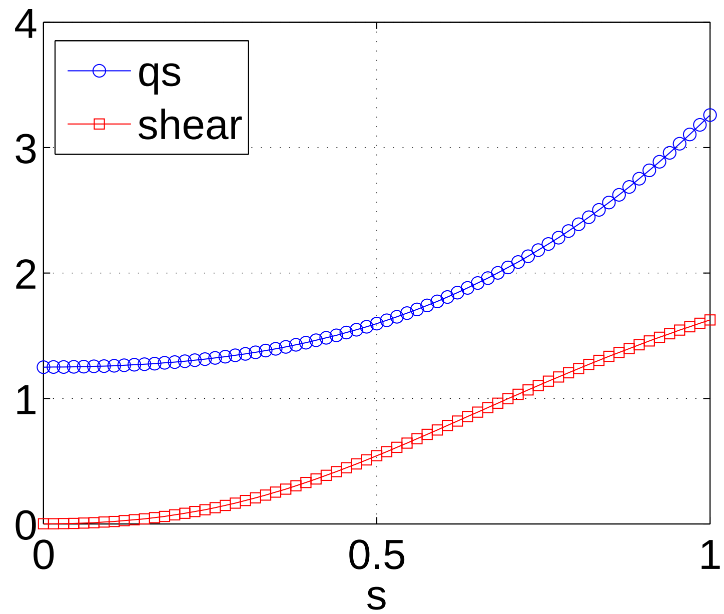
Global linear gyrokinetic theory (16)

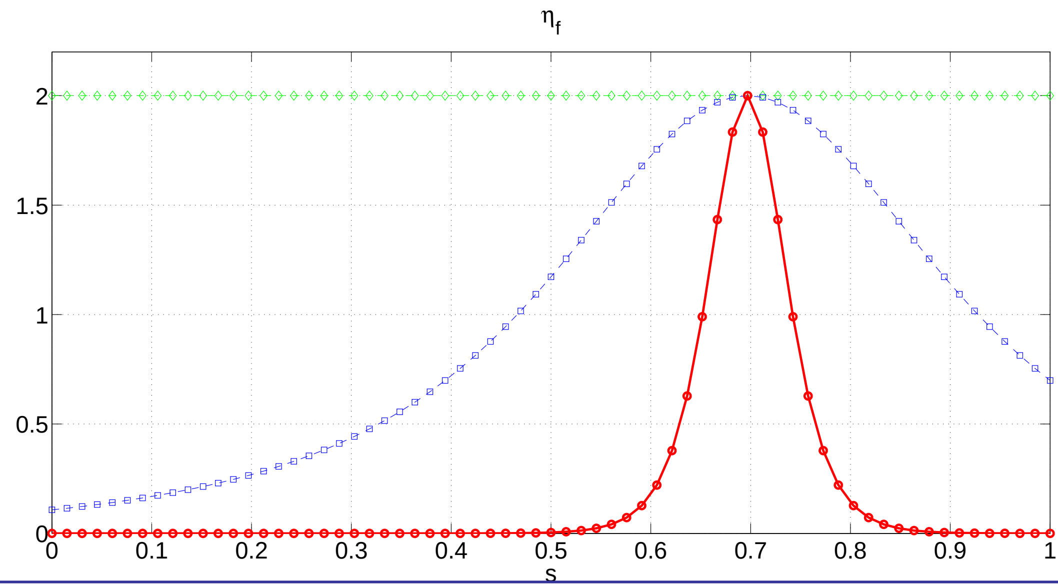
- FLR effects to all orders in $k_\perp \rho_{L,j}$ are retained for all species!
- Shafranov shift and finite β effects are included, i.e. $(\varphi, A_{||}, A_\perp)$ fluctuations. Only electrostatic case without Shafranov shift was shown here.
- The model in its final form is solved numerically in the code EM-GLOGYSTO.
- Code is MPI based and runs on 15-20 nodes. Recently a portable version based on FFTW has been developed.
- Code was developed at Lausanne and later in India.

linear theory - results (1)

- Equilibrium profiles for ITG modes with nonadiabatic thermal ions/electrons (top row) + Fast particles (bottom row)

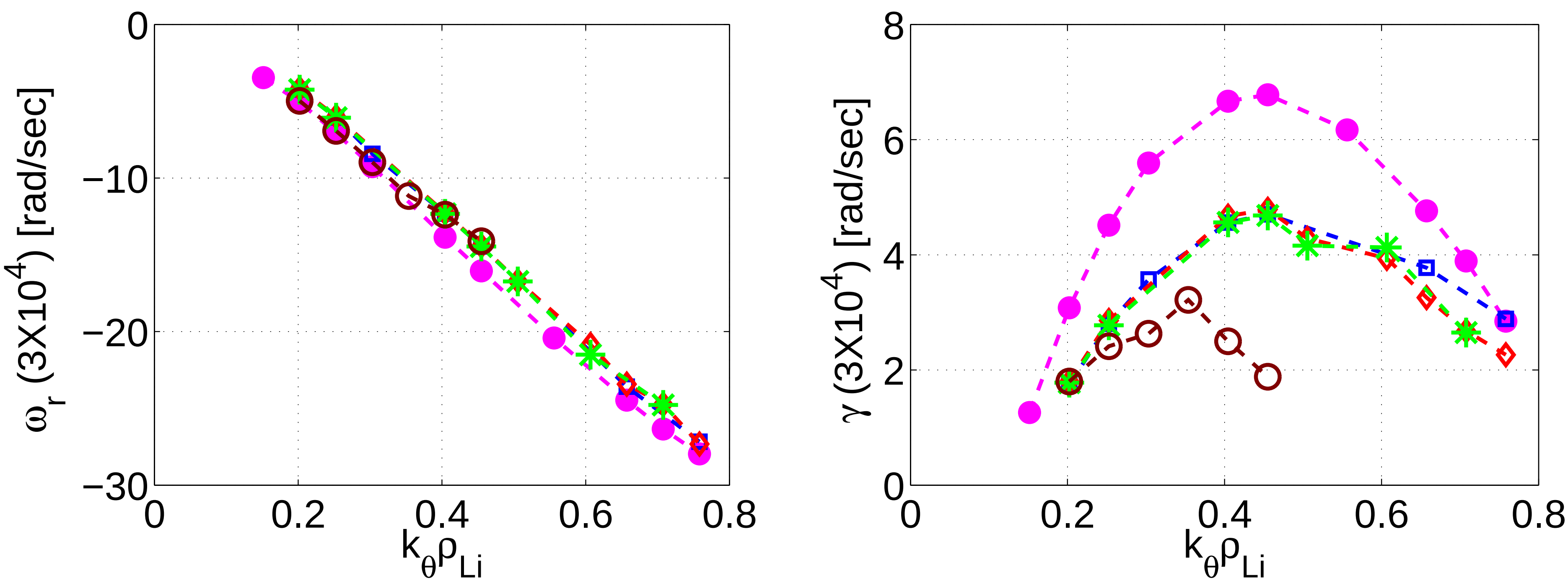






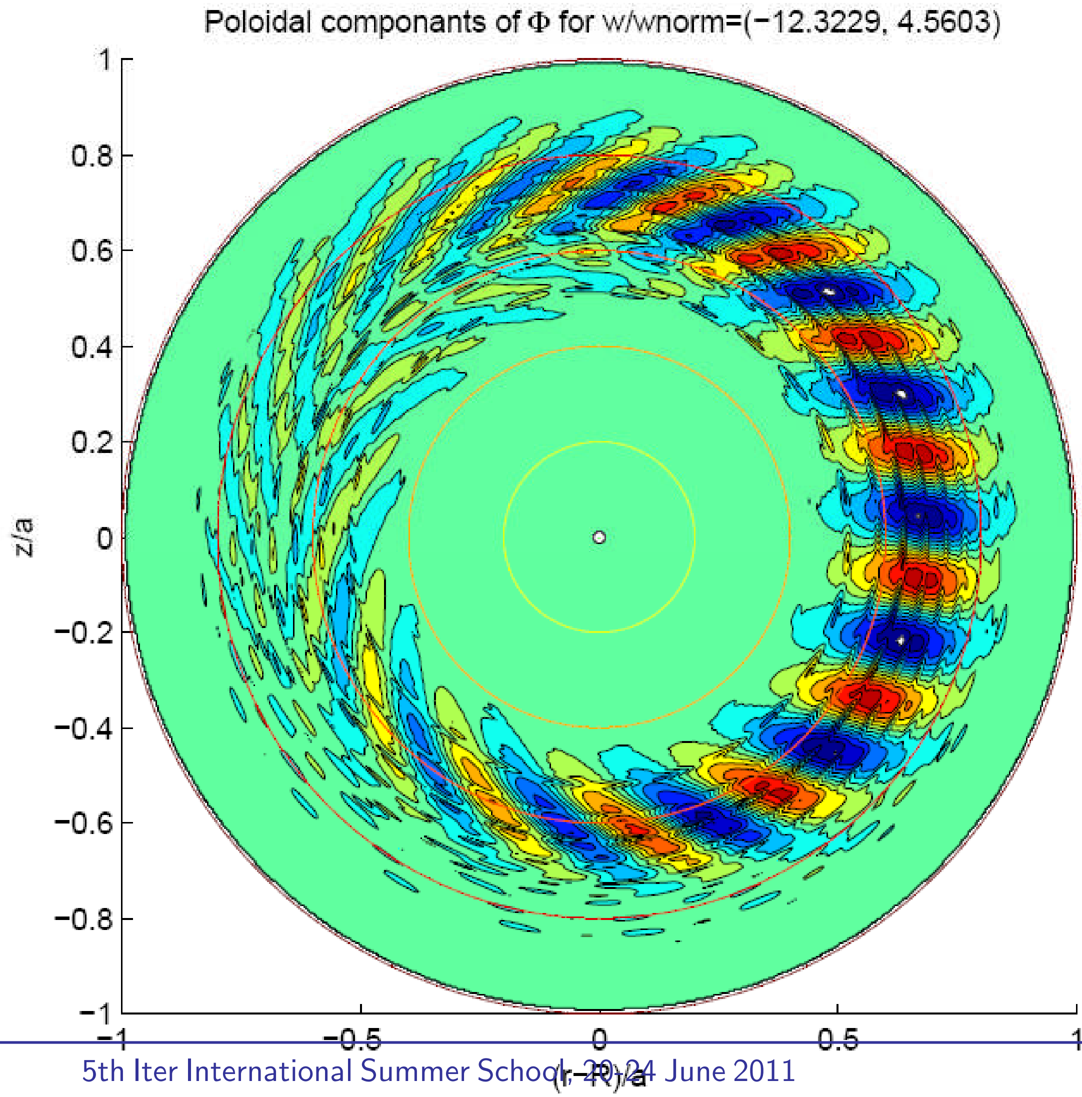
linear theory - results (2)

- The wavenumber scan for the mode frequency ω_r (left panel) and growth rate γ (right panel) of the ITG mode is presented for the cases, viz., 1. without energetic ions (magenta line), 2. with singly charged energetic ions with flat η profile (green line), 3. peaked η profile (red line), 4. same η profile (blue line) as the thermal ions and 5. with energetic He ions (brown line).
- For single charged ion $\frac{m_f}{m_i}$ =1.0, z_f =1.0, $\frac{T_f}{T_i}$=20, $\frac{n_f}{ne}$=0.1
- For He ion $\frac{m_f}{m_i}$ =2.0, z_f =2.0, $\frac{T_f}{T_i}$=2 0, $\frac{n_f}{ne}$=0.06.



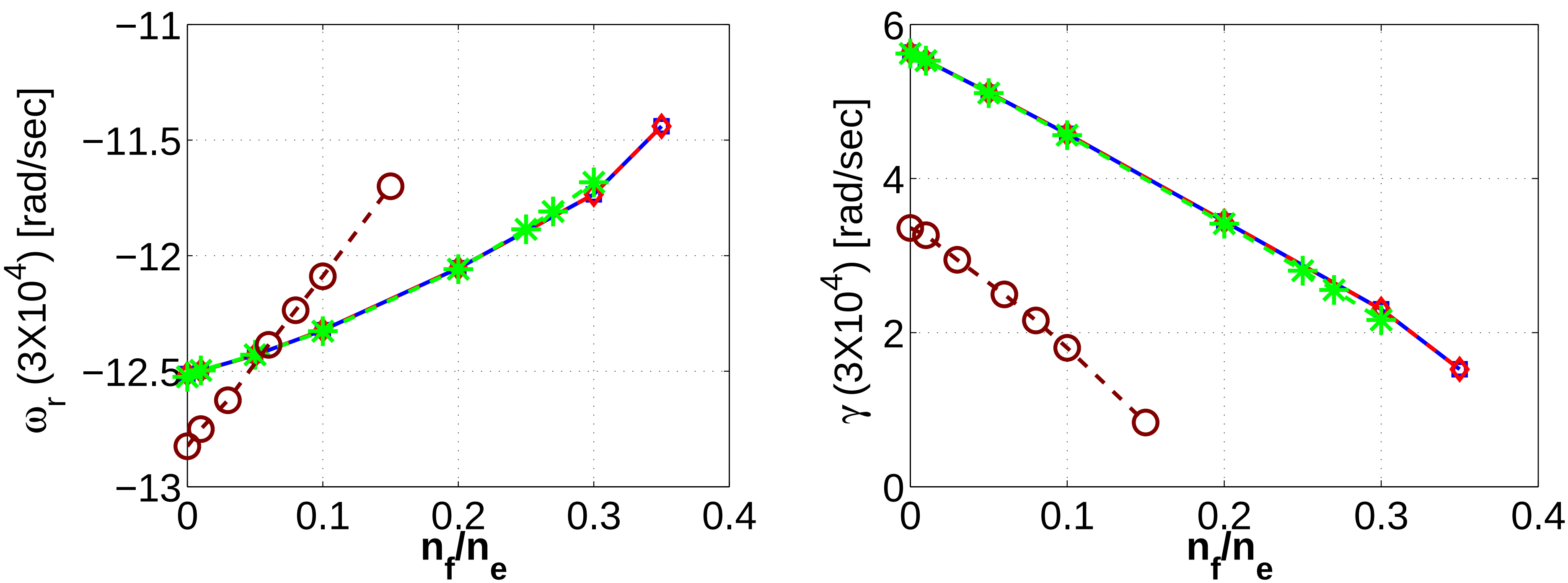
linear theory - results (3)

- Eigenmode for global ITG with nonadiabatic electrons and fast ions for $n = 8$ and $k_\theta \rho_{Li} = 0.4$ for fast ion profiles same as the thermals.



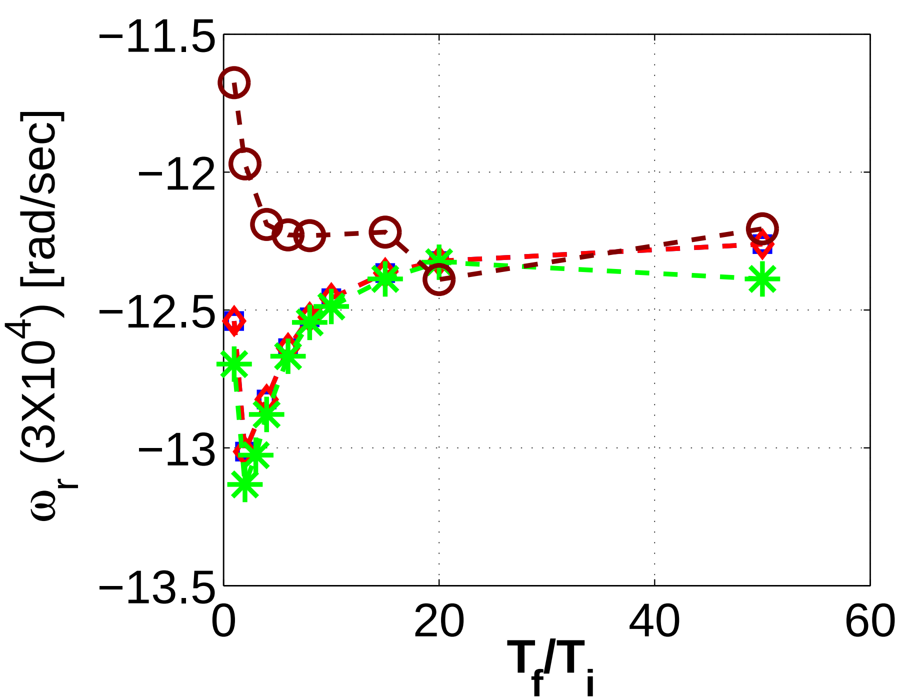
linear theory - results (4)

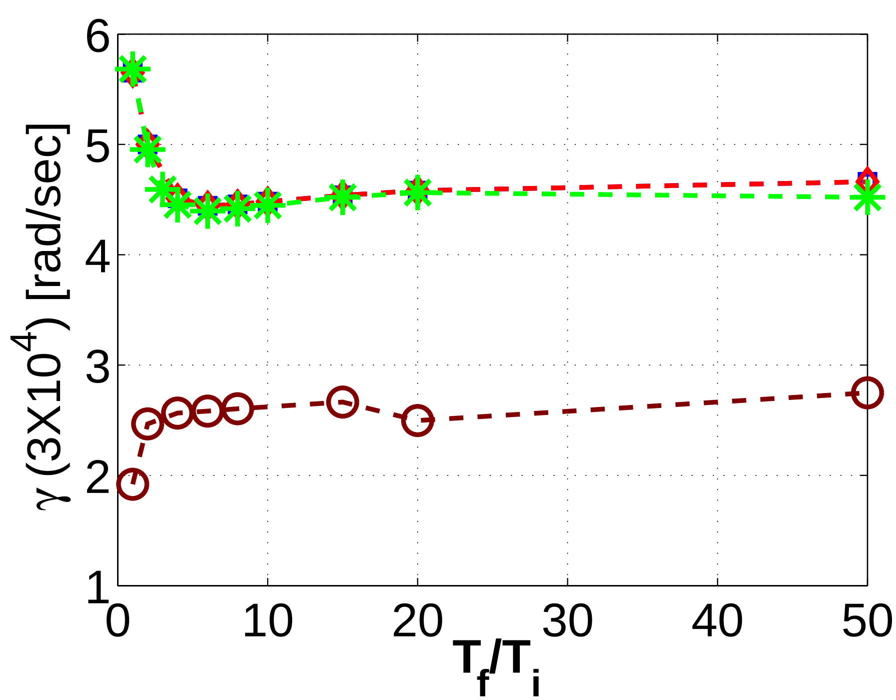
- The mode frequency ω_r and the growth rate γ as a function of density fraction of the energetic ions compared to background density of electron for the mode with n=8, $k_\theta \rho_{Li} = 0.4$ for the cases, viz., 1. with singly charged energetic ions with flat η profile (green line), 2. peaked η profile (red line), 3. same η profile (blue line) as the thermal ions and 4. with energetic He ions (brown line).



linear theory - results (5)

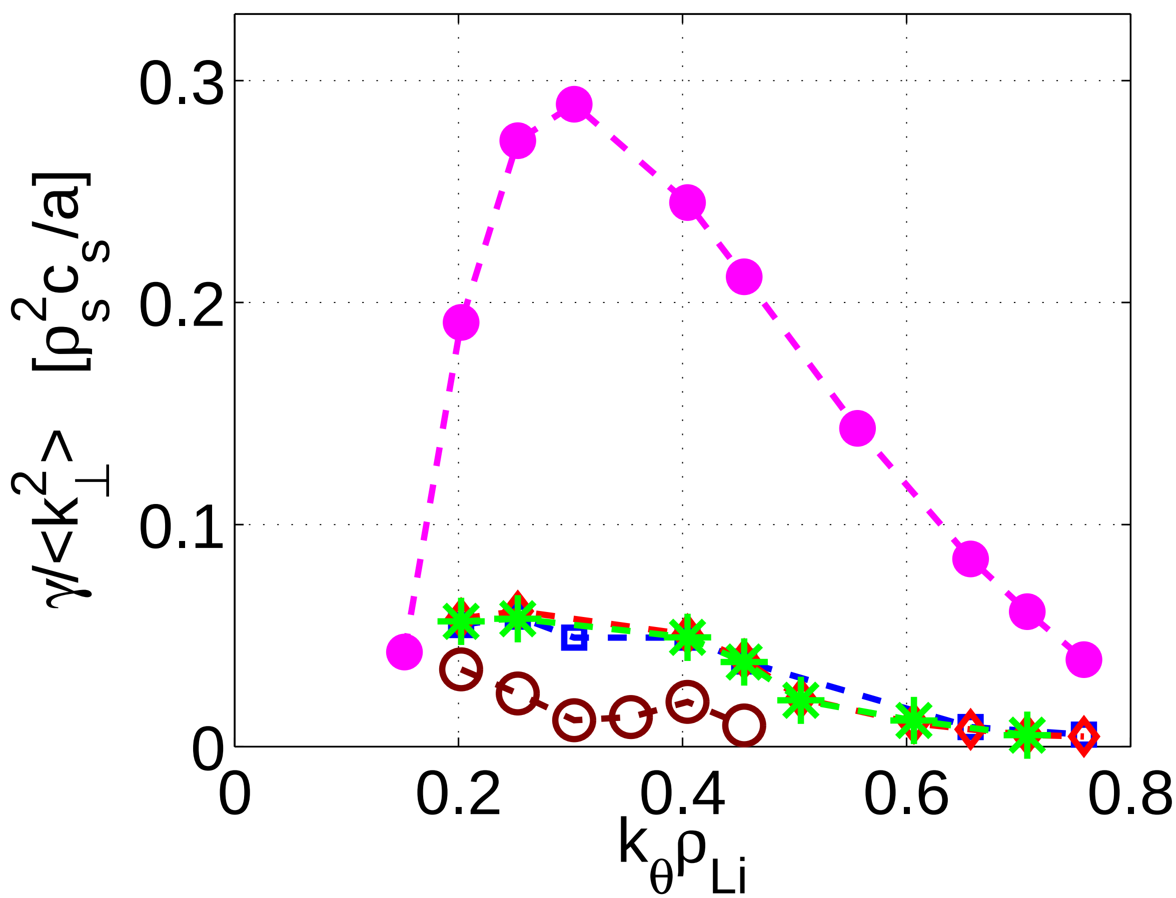
- The mode frequency ω_r and the growth rate γ are plotted as a function of density fraction of the energetic ions compared to background density of electron for the mode with n=8, $k_\theta \rho_{Li} = 0.4$ for the cases, viz., 1. with singly charged energetic ions with flat η profile (green line), 2. peaked η profile (red line), 3. same η profile (blue line) as the thermal ions and 4. with energetic He ions (brown line).





linear theory - results (6)

- A mixing length based estimation of transport in gyroBohm units is presented for the cases, viz., 1. without energetic ions (magenta line), 2. with singly charged energetic ions with flat η profile (green line), 3. peaked η profile (red line), 4. same η profile (blue line) as the thermal ions and 5. with energetic He ions (brown line)



linear theory - some comments(1)

- Global, fully gyrokinetic, linear stability formulation including nonadiabatic electrons and nonperturbative fast particle component was shown.
- In general, ITGs appear to be stabilized by energetic components. Electromagnetic effects are expected to further stabilize ITGs (not shown here).
- Spatial profile of fast particle drive seems unimportant.
- It appears that in an MHD stable equilibrium with fast particle population, ITGs would be completely benign! Then why study nonlinear ITGs? - Answer is : Don't know!

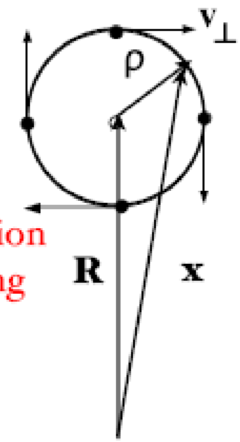
Nonlinear theory - issues for fast particles

- Using fast particles as "passive particles" i.e, their electromagnetic fields do not alter the background dynamics, is it possible to answer the following questions:
  - ▹ How does the fast particle transport get affected by global gyrokinetic turbulence (ITG/TEM) ?
  - ▹ Is the diffusion of fast particles dependent on its own energy (or temperature)?
  - ▹ Is the diffusion normal for all energies and system sizes?
  - ▹ As the turbulent drive increases (say larger and larger η), does this affect energetic particle transport?
- Can these fast particles be treated as "active particles" so that the dynamics becomes self-consistent?
  - ▹ At least the linear nonpertubative global calculations imply that ITGs can be made benign.

Governing Low-Frequency Gyrokinetic Vlasov-Maxwell Equations

$$\frac{dF_\alpha}{dt} \equiv \frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt}\cdot\frac{\partial F_\alpha}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt}\frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

GK Vlasov Equation



$$\left[\begin{array}{l} \dfrac{d\mathbf{R}}{dt} = v_\parallel \mathbf{b}^* + \dfrac{v_\perp^2}{2\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times \nabla ln B_0 - \dfrac{c}{B_0}\nabla\bar{\phi}\times\hat{\mathbf{b}}_0 \\ \dfrac{dv_\parallel}{dt} = -\dfrac{v_\perp^2}{2}\mathbf{b}^*\cdot\nabla ln B_0 - \dfrac{q_\alpha}{m_\alpha}\left(\mathbf{b}^*\cdot\nabla\bar{\phi} + \dfrac{1}{c}\dfrac{\partial \bar{A}_\parallel}{\partial t}\right) \\ \mu_B = \dfrac{v_\perp^2}{2B_0}\left(1 - \dfrac{mc}{e}\dfrac{v_\parallel}{B_0}\hat{\mathbf{b}}_0\cdot\nabla\times\hat{\mathbf{b}}_0\right) \approx const. \end{array}\right]$$

Coordinates Transformation and Gyrophase Averaging

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0\cdot\nabla)\hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla\times\bar{\mathbf{A}}}{B_0} \qquad \begin{pmatrix}\bar{\phi}\\ \bar{\mathbf{A}}\end{pmatrix}(\mathbf{R}_{\alpha j}) = \left\langle\begin{pmatrix}\phi\\ \mathbf{A}\end{pmatrix}(\mathbf{x}_{\alpha j})\right\rangle_\varphi$$

$$\frac{\rho_s^2}{\lambda_D^2}\nabla_\perp^2\phi(\mathbf{x}) = -4\pi(1-\rho_i^2\nabla_\perp^2)\rho(\mathbf{x}) \qquad \rho(\mathbf{x}) = \sum_\alpha q_\alpha \sum_{j=1}^N \langle\delta(\mathbf{x}-\mathbf{x}_{\alpha j})\rangle_\varphi$$

GK Poisson's Equation with Pede Approximation

$$\nabla^2\mathbf{A}(\mathbf{x}) - \frac{1}{v_A^2}\frac{\partial^2 \mathbf{A}_\perp(\mathbf{x})}{\partial t^2} = -\frac{4\pi}{c}\mathbf{J}(\mathbf{x})$$

GK Ampere's Law

$$\mathbf{J}(\mathbf{x}) = \sum_\alpha q_\alpha \sum_{j=1}^N \left[(\mathbf{v}_{\parallel\alpha j} + \mathbf{v}_{d\alpha j})\langle\delta(\mathbf{x}-\mathbf{x}_{\alpha j})\rangle_\varphi + \langle\mathbf{v}_{\perp\alpha j}\delta(\mathbf{x}-\mathbf{x}_{\alpha j})\rangle_\varphi\right]$$

$$\mathbf{v}_d \equiv v_\parallel^2\hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0\cdot\nabla)\hat{\mathbf{b}}_0 + \frac{v_\perp^2}{2}\hat{\mathbf{b}}_0\times\nabla ln B_0$$

[Lee, Dubin et al., Hahm, Brizard, Qin et al, PPPL]

Nonlineary theory : Standard Delta-f scheme

Perturbative Simulation of ITG modes with Adiabatic Electrons

$$F = F_0 + \delta f \qquad w \equiv \frac{\delta f}{F}$$

[Parker and Lee, 1993; Lin, Tang, Lee, 1995]

$$\delta f = \sum_{j=1}^{N} w_j \delta(\mathbf{R} - \mathbf{R}_j)\delta(\mu - \mu_j)\delta(v_\parallel - v_{\parallel j}) \qquad \mu = \frac{v_\perp^2}{2}$$

$$\frac{d\mathbf{R}}{dt} = \mathbf{u} + \mathbf{v}_d + \mathbf{v}_{E\times B} \qquad \mathbf{u} = v_\parallel \hat{\mathbf{b}}_0$$

$$\frac{du}{dt} = -\mathbf{b}^* \cdot \left(\frac{v_\perp^2}{2} \nabla ln B_0 + \nabla \bar{\phi} \right)$$

$$\mu_B = \frac{v_\perp^2}{2B_0} \approx const.$$

$$\frac{dw}{dt} = -(1-w)\left[(\mathbf{v}_{E\times B} + \mathbf{v}_d) \cdot \kappa \hat{\mathbf{r}} + \frac{T_e}{T_i}(\mathbf{u} + \mathbf{v}_d) \cdot \nabla \bar{\phi} \right]$$

Turbulent Drive ($\mathbf{v}_{E\times B}$) — Neoclassical Drive ($\mathbf{v}_d$)

$$\mathbf{b}^* \equiv \hat{\mathbf{b}}_0 + v_\parallel \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla)\hat{\mathbf{b}}_0 \qquad \mathbf{v}_{E\times B} = -\nabla \bar{\phi} \times \hat{\mathbf{b}}_0$$

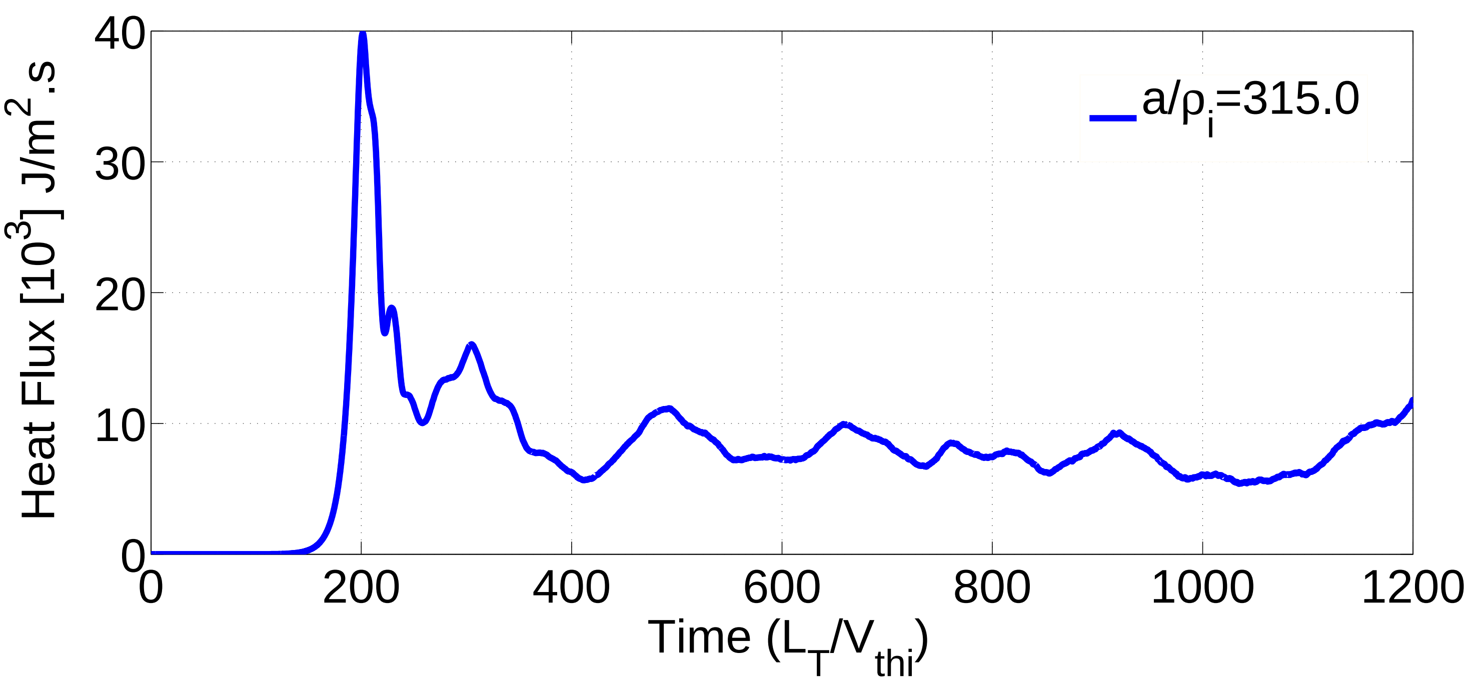
$$\mathbf{v}_d \equiv v_\parallel^2 \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla)\hat{\mathbf{b}}_0 + \frac{v_\perp^2}{2}\hat{\mathbf{b}}_0 \times \nabla ln B_0$$

$$\nabla_\perp^2 \phi = -4\pi e \delta n_i \qquad k_\parallel \neq 0 \qquad \delta n_i = \int \delta f_i du d\mu$$

$$\nabla_\perp^2 \phi - \phi = -4\pi e \delta n_i \qquad k_\parallel = 0$$

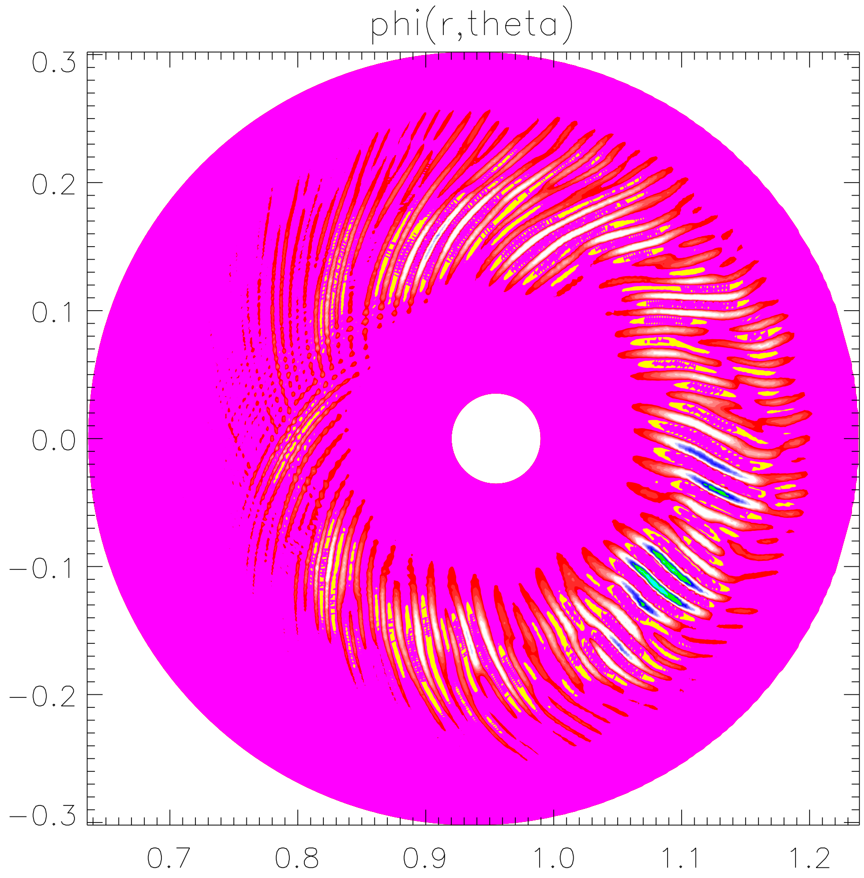
Nonlinear gyrokinetics- results (1)

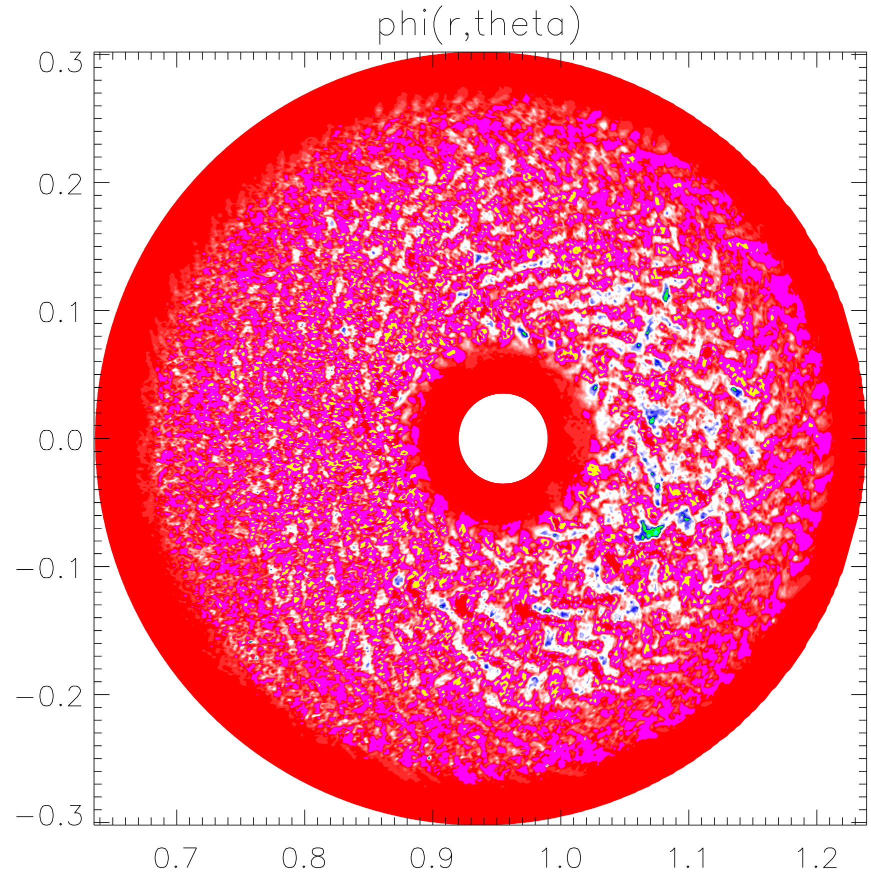
- For simulations presented here thermal species are represented by 0.12 Billion particles. Energetic particle are represented by 6.7 Million particles per energy value. 1024 compute nodes were used in parallel using GTS code.
- Figure shown depicts the time history of the thermal ion heat flux arising from the ITG turbulence with system size $a/\rho_i = 315$. The heat flux is calculated using the relation $Q_i = \int d^3v \frac{1}{2} v^2 v_E \delta f$, where v is particle velocity, v_E is the radial component of gyro-averaged $E \times B$ drift and δf is the perturbed distribution function, and recorded at $r = 0.5a$ at every time step.



Nonlinear gyrokinetics- results (2)

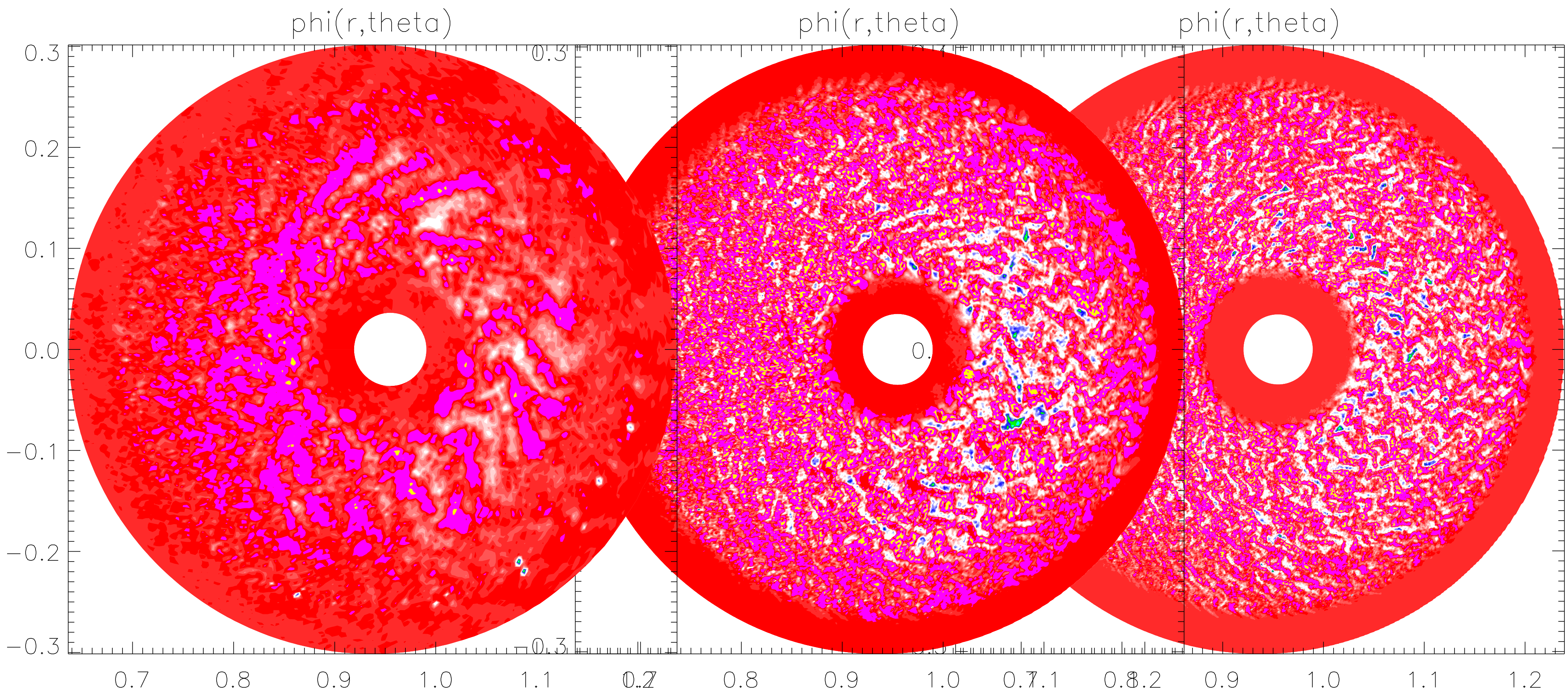
- 2D potential structures due to ITG in at t=200, linear regime (left) and t=1100, nonlinear regime (right) of the turbulence for $a/\rho_i = 315$.
- The linear structure gets broken due to generation of self-consistent shear flow called Zonal flows in the poloidal direction.





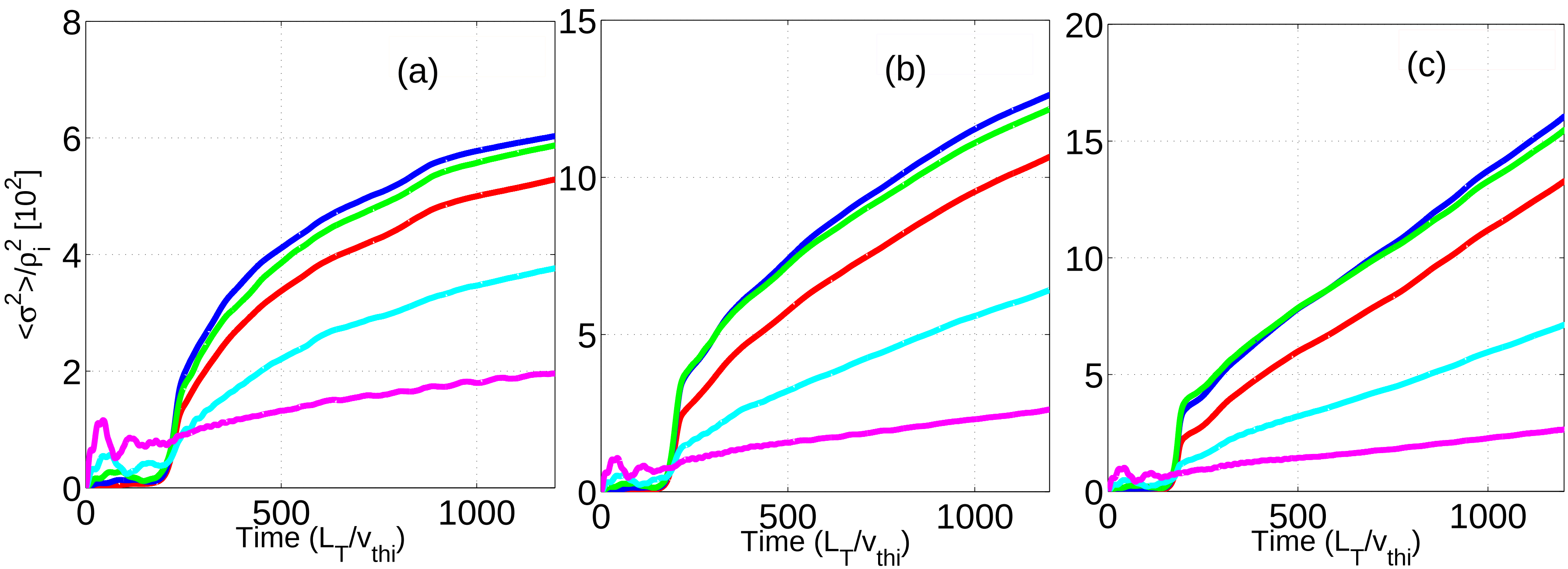
Nonlinear gyrokinetics- results (3)

- Comparision of 2D potential structures due to ITG t=1100 in its nonlinear regime for $a/\rho_i = 157$ (left), $a/\rho_i = 315$ (middle),$a/\rho_i = 500$ (right)
- 2D structures or Eddie size appears to decrease from left to right. Actually the turbulence decorrelation length or Eddy size is INDEPENDENT of the system size! There are simply more eddies in the larger system than in the smaller one.



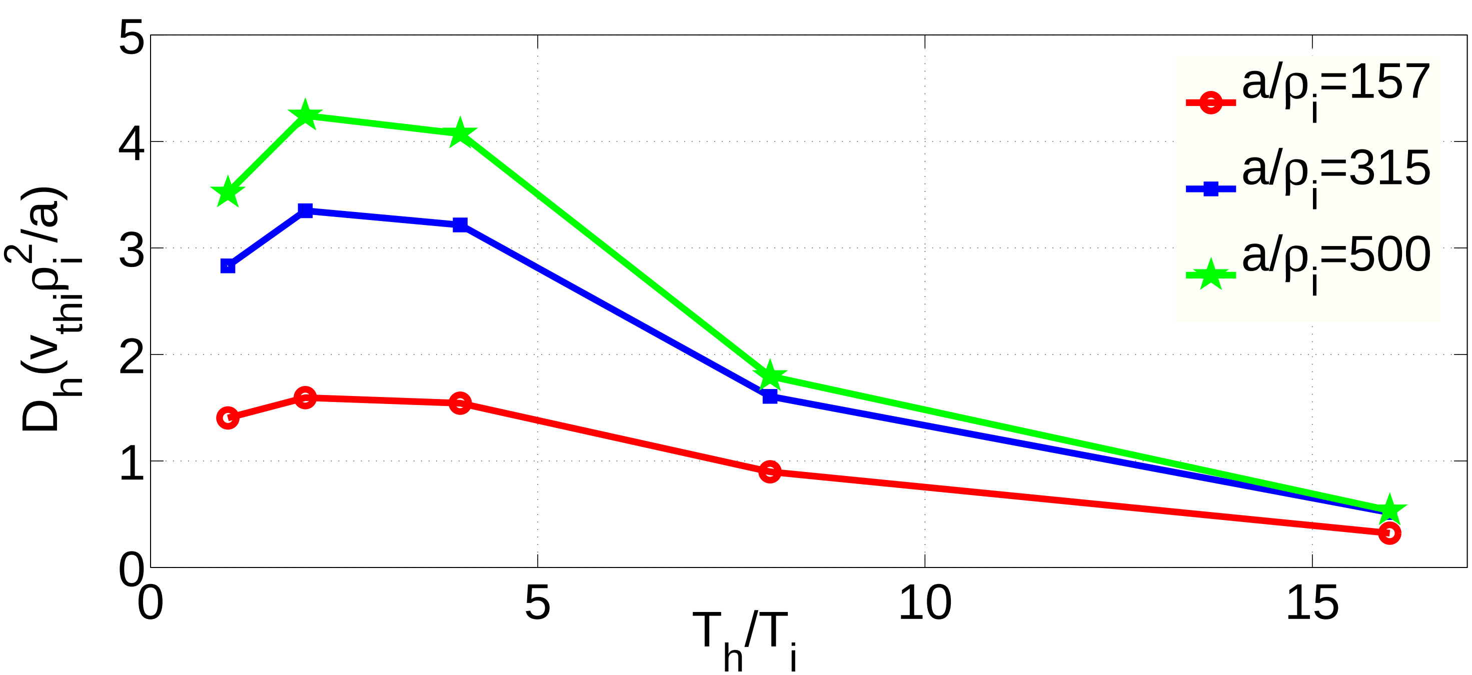
Nonlinear gyrokinetics- results (4)

- Passive fast particles or test particles of various energy values ϵ with uniform pitch angle is introduced at $r = 0.5\ a$ for each system size: $a/\rho = 157$ (left), $a/\rho = 315$ (middle), $a/\rho = 500$ (right)
- Mean square displacement (MSD) of the hot ions defined as $< \sigma^2(\varepsilon, t) >= \frac{1}{N}\sum_{i=1}^{i=N}(r_i(\varepsilon, t) - r_i(\varepsilon, 0))^2$ where, N is the total number of particles of hot ions with energy ε, $r_i(\varepsilon, t)$ and $r_i(\varepsilon, 0)$ are, respectively, the radial positions of the ith hot ion with energy ε at time t and $t = 0$. $T_h = T_i$ (red), $T_h = 2T_i$ (blue), $T_h = 4T_i$ (green), $T_h = 8T_i$ (cyan), $T_h = 16T_i$ (magenta).



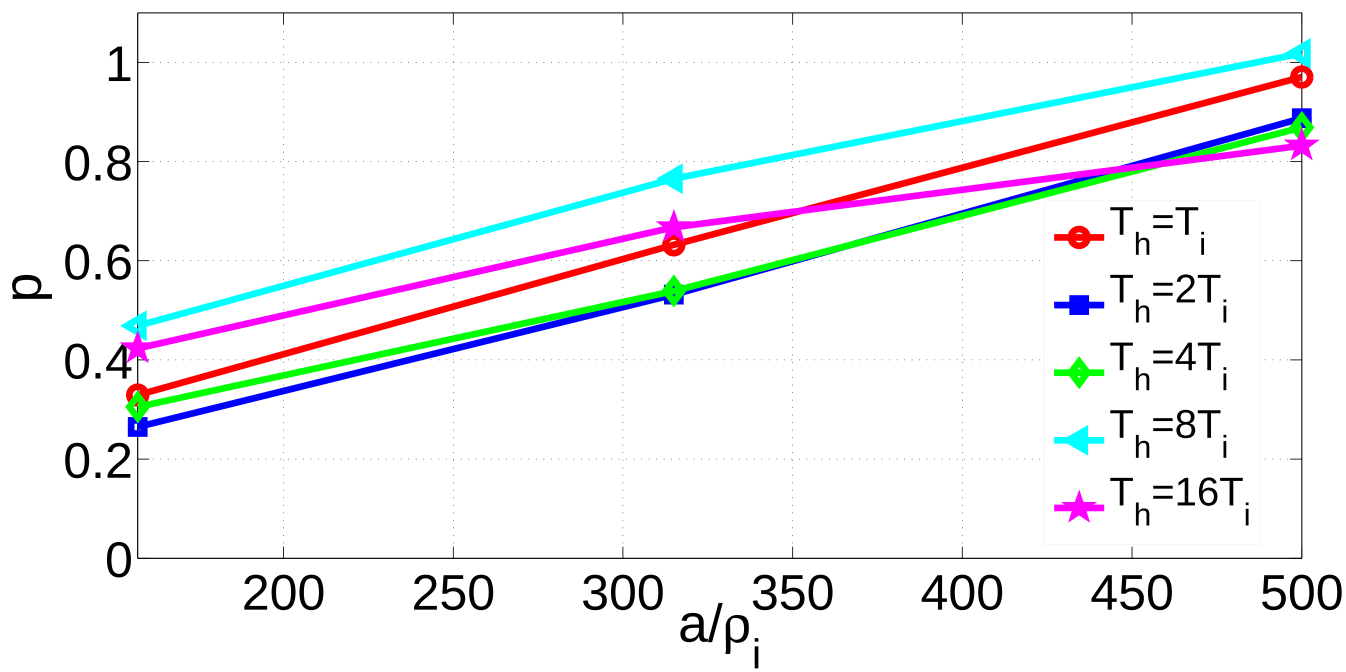
Nonlinear gyrokinetics- results (5)

- Diffusion is properly defined only at late times. One could define $D_h = \delta < \sigma^2 > / \delta t = (< \sigma^2(t_2) > - < \sigma^2(t_1) >)/(t_2 - t_1)$ between two late times t_1 and t_2.
- D_h for 3 system size and different energies shows that diffusion increases with system size for thermals whereas it does not increase so much with system size for hot particles and nearly independent of the system size at high energies.



Nonlinear gyrokinetics- results (6)

- MSD $< \sigma^2 >= At^p$ where p is the exponent. $p = 1$ for normal diffusion, $p < 1$ for subdiffusion $p > 1$ for superdiffusion and $p = 2$ motion is ballistic.
- The value of p, calculated from the slope of $log < \sigma^2 >$ versus $log\ t$ between time $t = 900 L_T / v_{thi}$ to $t = 1200 L_T / v_{thi}$
- Except for large system sizes, diffusion is mostly subdiffusive !



Nonlinear gyrokinetics- results (7)

- A Probabality density function (a PDF) was constructed with 200 bins and more than 1000 particles per bin.
- Probability density function (PDF) of particles at $t = 1200$.
- Standard deviation σ, skewness s and kurtosis k, defined respectively, as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

,

$$s = \sqrt{N}\frac{\sum_{i=1}^{N}(x_i - \bar{x})^3}{\left\{\sum_{i=1}^{N}(x_i - \bar{x})^2\right\}^{3/2}}$$

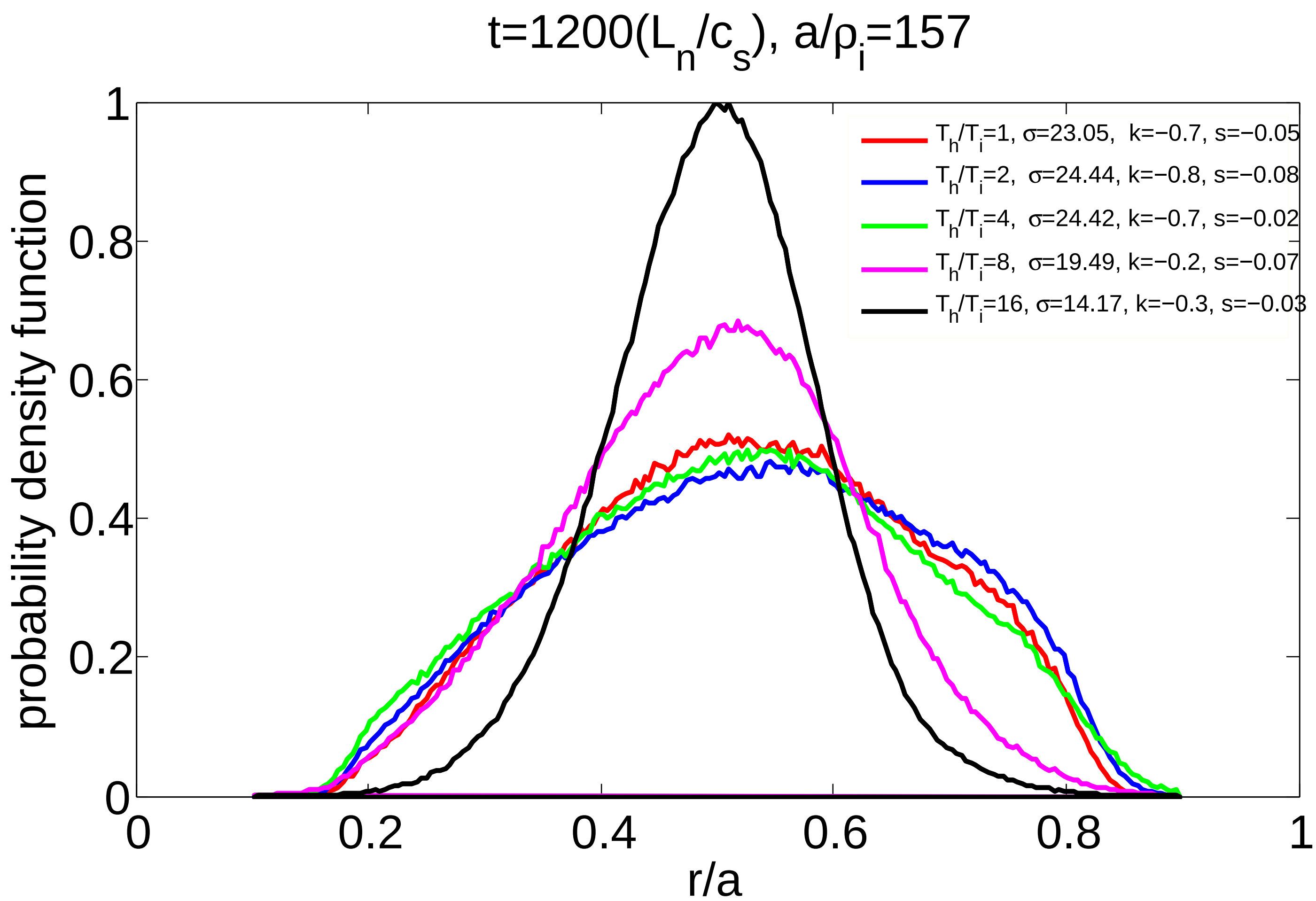
and

$$k = N\frac{\sum_{i=1}^{N}(x_i - \bar{x})^4}{\left\{\sum_{i=1}^{N}(x_i - \bar{x})^2\right\}^{2}} - 3$$

- Kurtosis (relative peakedness), Skewness (measure of symmetry about the Mean). For Gaussian, both should be zero as defined above.
- Deviations of these quantities from zero implies non-Gaussianity and non-normal diffusion.

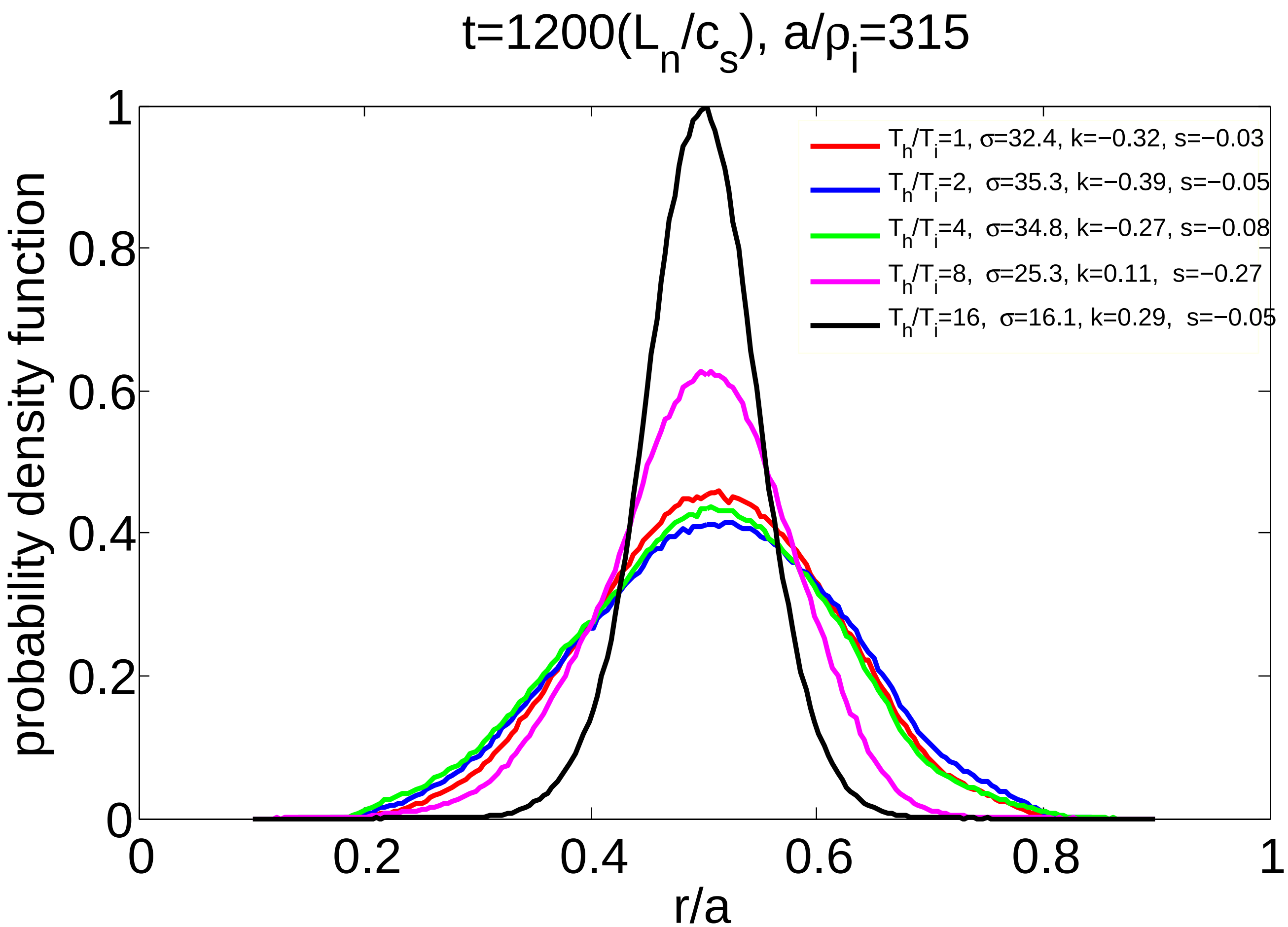
Nonlinear gyrokinetics- results (8)

- PDF at late times, indicates non-Gaussianity for small system size for low energies.



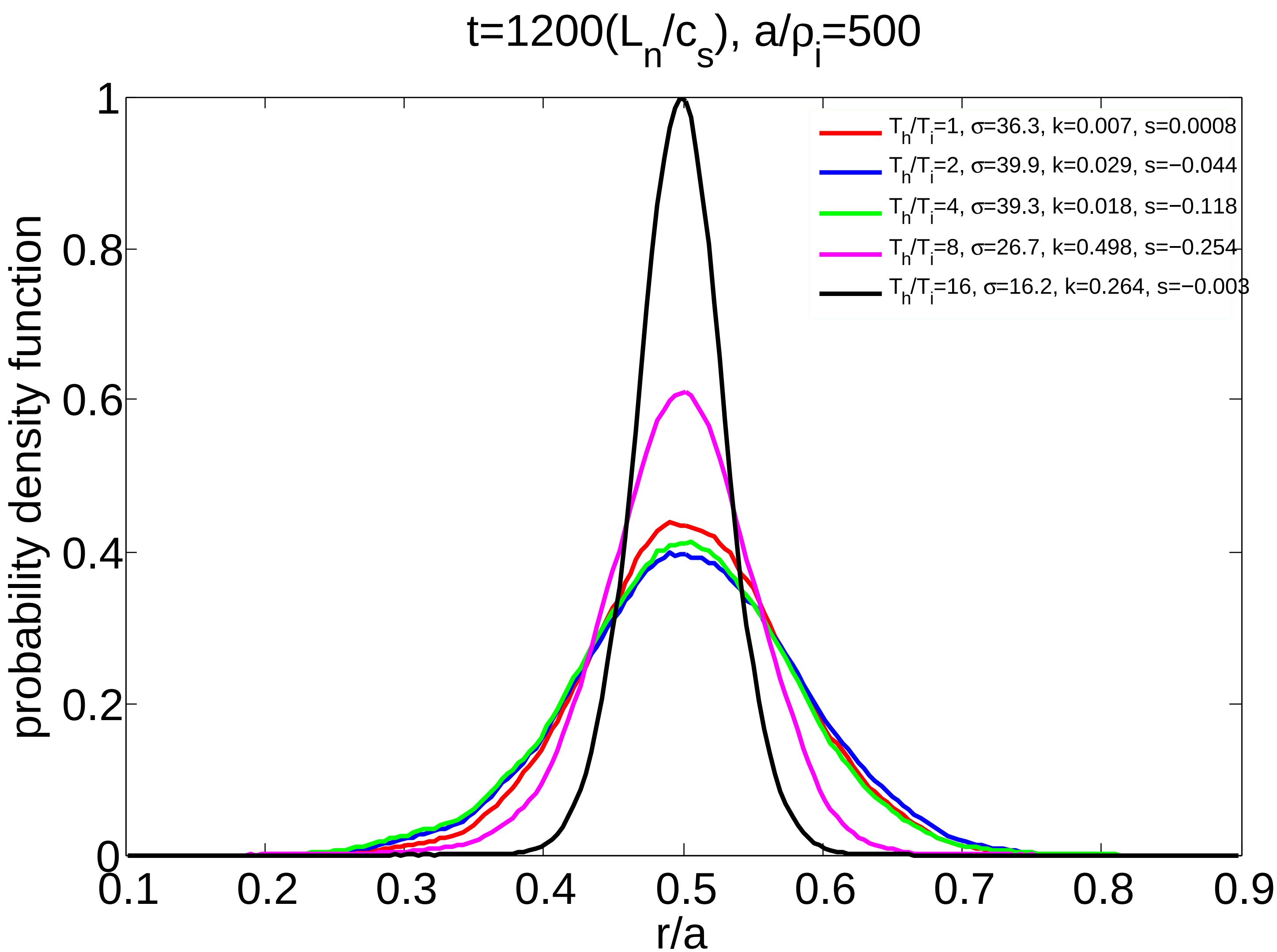
Nonlinear gyrokinetics- results (9)

- PDF at late times, indicates non-Gaussianity for intermediate system size (315) for low energies.



Nonlinear gyrokinetics- results (10)

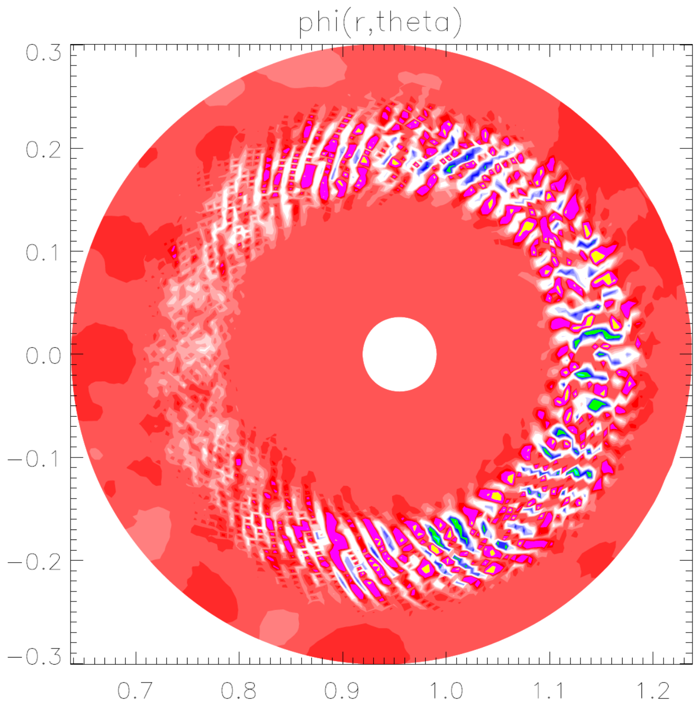
- PDF at late times, indicates Gaussianity for large system size (315) for all energies.

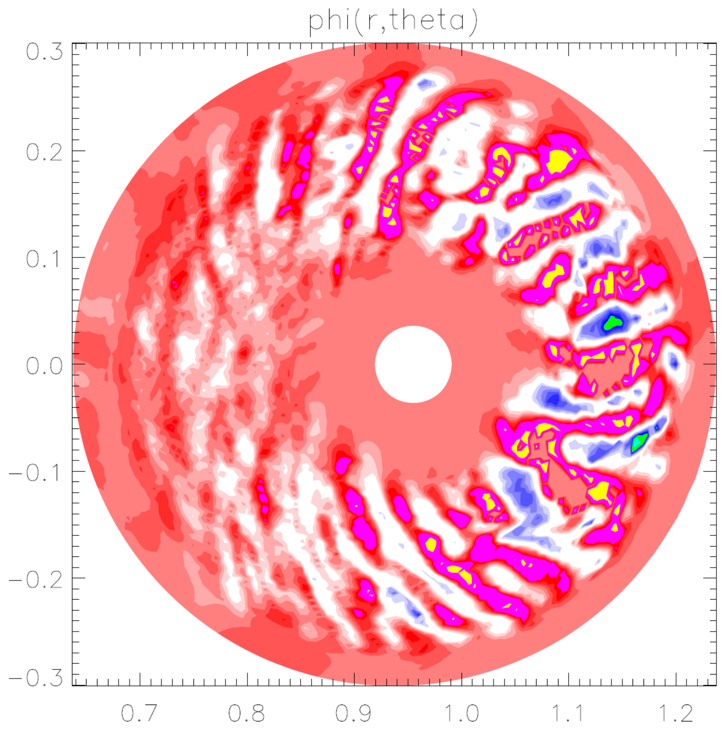


Nonlinear gyrokinetics- results (11)

- Nonlinear quasi-2D structures or turbulent eddies result from nonlinear ITG turbulence.
- These electrostatic structures tend to "rotate" or "trap" particles in their ExB motion
- Eddy-eddy interaction "detraps" the particle which then scatter away into regions between eddies.
- Typical eddy sizes or turbulent decorrelation length scale is between $7 - 20\ \rho_{Li}$ (This is a numerical estimate from two different studies]
- For particles considered here Larmor radii is about $1 - 6\rho_{Li}$, hence both thermals and energetic particle can interact with the eddies.
- Eddy sizes are independent of system size and hence there would be relatively more such structures in large system size than a smaller one.
- In small systems, particles tend to spend, on an average more time in "trapped" regions (or ellipic regions) then in regions inbetween (or hyperboloic regions) and hence sub-diffuse.
- In large systems, particles tend to spend, on an average more time in "detrapped" regions (or hyperbolic regions) then in regions in the ellipic regions because of larger eddy-eddy interactions
- High energy particles possibly orbit average strongly than low energy particles.

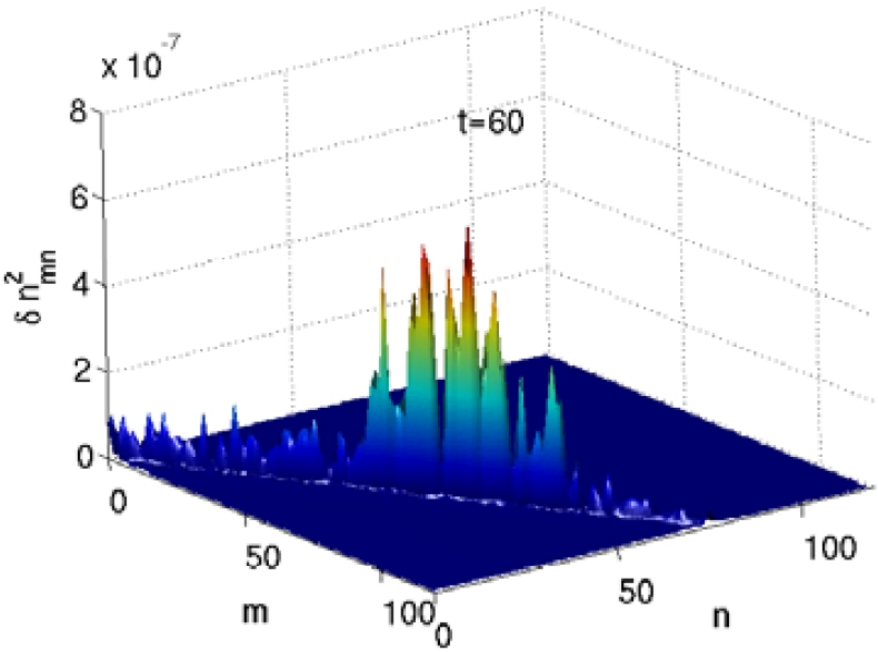
- 2D potential structures due to TEM turbulence at quasilinear regime (left) and nonlinear regime (right) of the turbulence for $a/\rho_i = 157$.
- Structures grow and "inverse cascade" ensues.

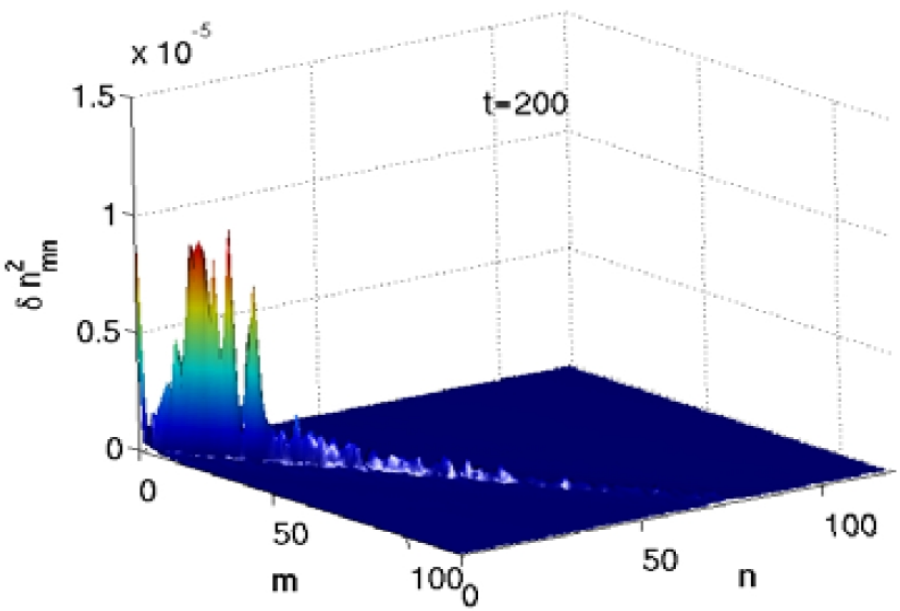




Nonlinear gyrokinetics- results (13)

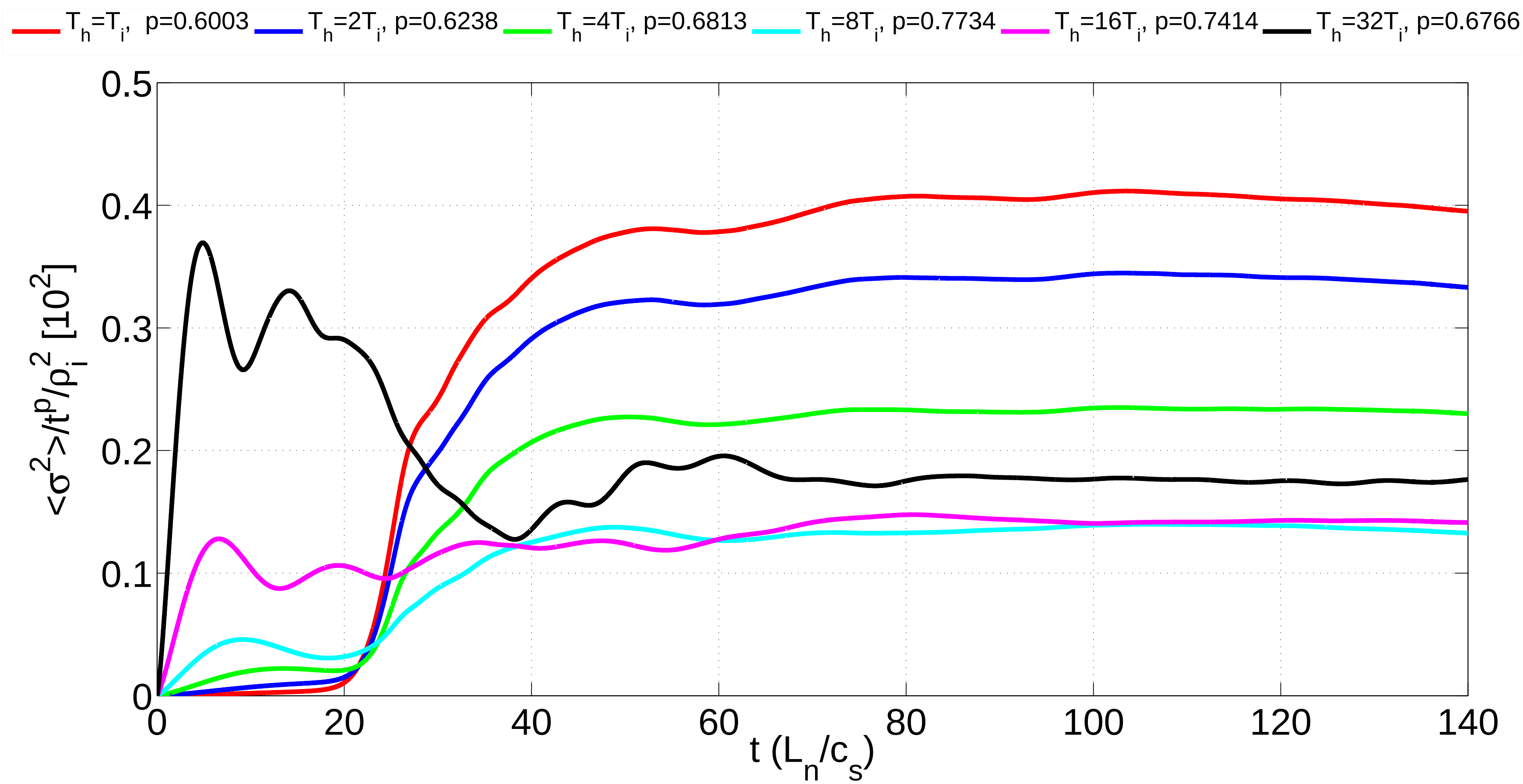
- Fourier analysis of the potential fluctuations in toroidal (n) and poloidal (m) direction yield the 2D power spectrum.
- In the quasilinear regime (left), the power spectrum peaks at $(m, n) \simeq (50, 50)$. At the later stages the power spectrum peaks at $(m, n) \simeq (4, 5)$ clearly showing energy going to large scales leading to formation of large scale eddies.





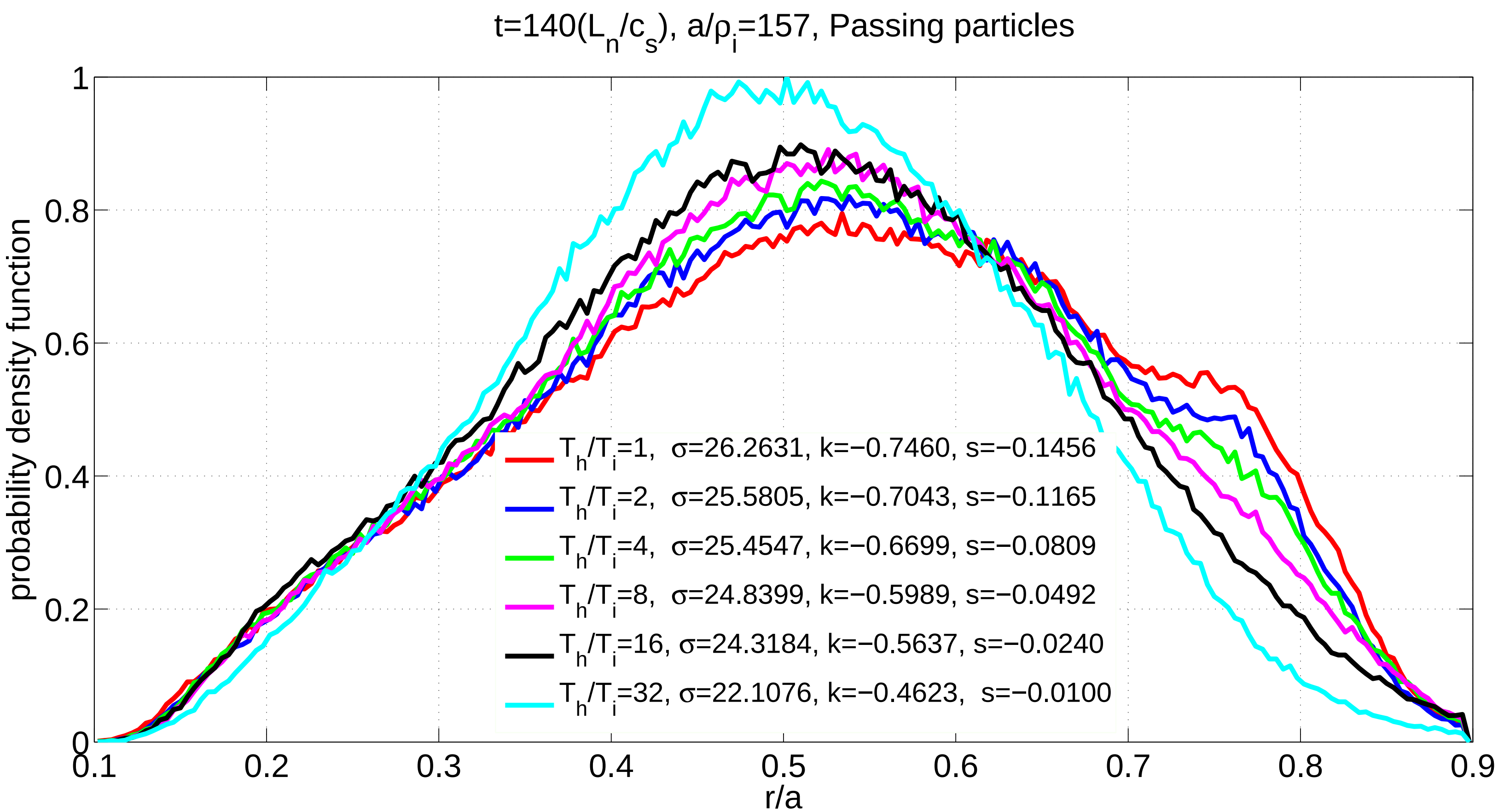
Nonlinear gyrokinetics- results (14)

- Test particle transport in TEM turbulence for small system size also shows strong subdiffusion.
- $< \sigma^2 > / t^p$ as function of time clearly shows saturation only for values of $p < 1$



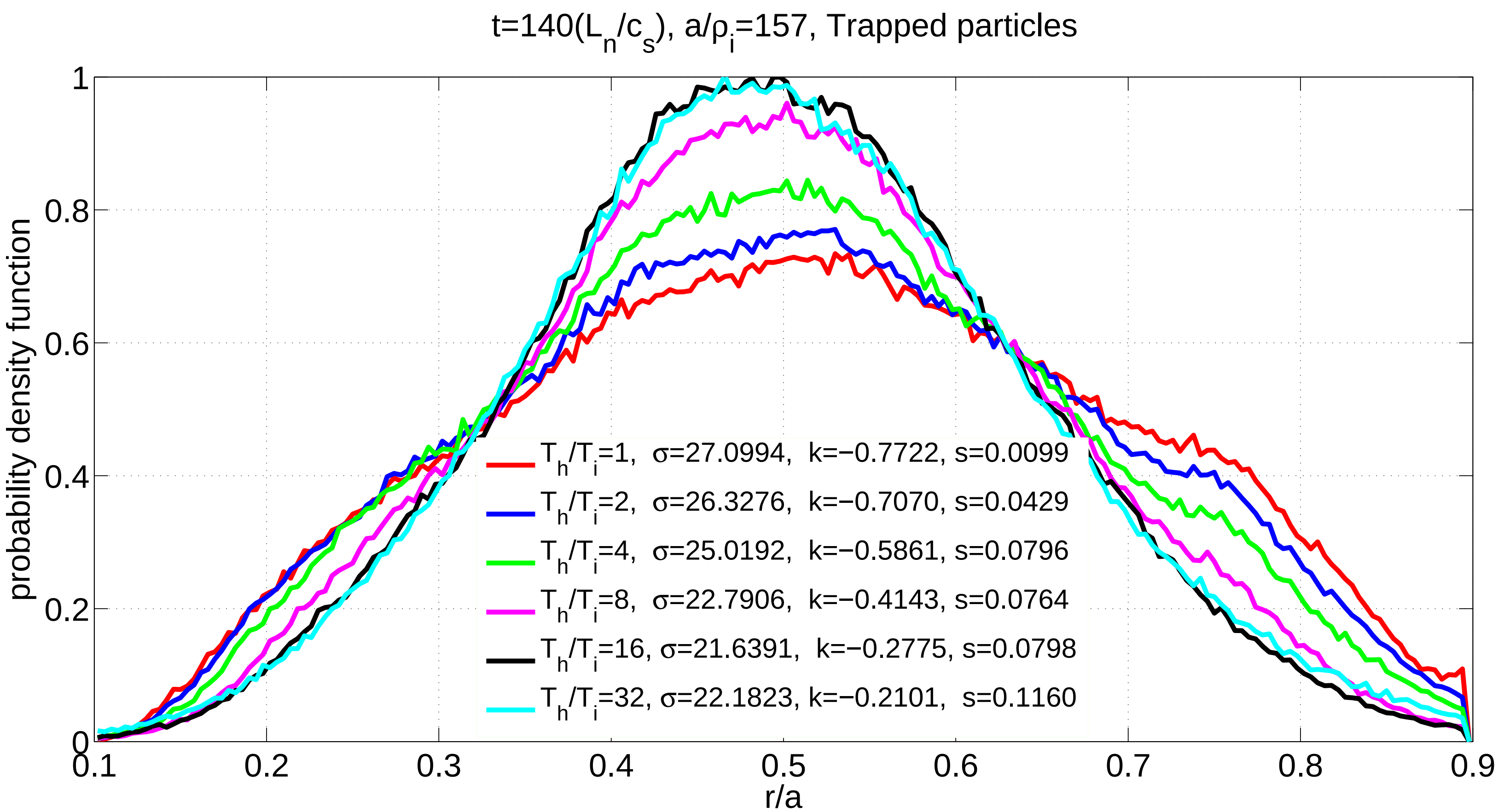
Nonlinear gyrokinetics- results (15)

- PDF shows nonGaussianity confirming the value of the exponent $p \simeq 0.6$ much less than 1.
- PDF of passing particles show deviations from Gaussianity.



Nonlinear gyrokinetics- results (16)

- PDF of trapped particles also show deviations from Gaussianity.



Some concluding comments (16)

- Nonperturbative linear global gyrokinetic stability analysis for ITG and fast particles show that the mode is rendered benign. For values of $n_f/n_e \simeq 0.1$ or more it is stabilized. Stabilization is stronger for Helium ions.
- Nonlinear global gyrokinetic simulations for ITG and passive fast particles indicate a clear transition from Subdiffusive regime at low system size to Diffusive regime at large system sizes for both the thermals as well as fast particles.
- PDFs constructed also corroborate.
- A heuristic explanation in terms of eddy trapping and detrapping is suggested.
- TEM turbulence for small system size also shows similar signature. (Large system size - work is ongoing)
- In general, for quasi-2D systems, the nature of diffusion (sub, normal, super) is probably determined by the ratio of effective "ellipic" area to the "hyperbolic" area $(> 1, 1, < 1)$!